ASPECTS OF SYMMETRY- IV

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ABSTRACT. In this issue, several aspects of symmetry in the arts, sciences and humanities are discussed, including the conservation laws in physics, electrical circuits, the works of J.L. Borges, M.C. Escher and A. Rublev, the golden ratio, handedness in embryonic development, sexual and facial attractiveness, social relations, and dancing.

CONTENTS

1. INTRODUCTION

Symmetry is a language with which many aspects of science and the arts can be expressed, appreciated, and eventually better understood and explained. The present issue introduces the reader to a number of those aspects found in several areas, including physics, electrical engineering, poetry, painting, aesthetics, embryonic development, sexual and facial attractiveness, social relations, and dancing.

2. CONSERVATION LAWS

Contributed by Anam Shaikh. Newton's laws of mechanics embodied symmetry principles, most particularly, the equivalence of internal frames (Galilean invariance). These symmetries ultimately implied conservation laws.

Although conservation laws were regarded to be of fundamental importance, they were regarded as consequences of the dynamic laws of nature rather than as consequences of the underlying symmetries. For

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example, Maxwell's equations exemplified both Lorentz invariance and gauge invariance. But the symmetries implicit in his equations of electrodynamics were not fully appreciated or even understood until decades later. With the advent of Einsteinian mechanics, symmetry principles became the primary feature of nature that constrains the allowable dynamic laws. Thus the transformation properties of the electromagnetic field were not to be derived from Maxwell's Equations, but rather were consequences of relativistic invariance, which inherently dictate the form of Maxwell's Equations. Einstein recognized the symmetry implicit in Maxwell's Equations and elevated it to a symmetry of space-time itself [1].

Symmetry and regularity. Progress in physics depends on the ability to separate the analysis of a physical phenomenon into two parts: arbitrary initial conditions and the laws of nature that summarize the regularities that are independent of the initial conditions. The laws are often difficult to discover since they can be hidden by the irregular initial conditions or by the influence of uncontrollable factors such as gravity friction or thermal fluctuations. Symmetry principles play an important role in the laws of nature as they summarize the regularities of the laws that are independent of the specific dynamics [1]. Invariance principles provide a structure and coherence to the set of events. Indeed, it is hard to imagine that much progress could have been made in deducing the laws of nature without the existence of certain symmetries. Without regularities embodied in the laws of physics one would be unable to make sense of physical events. Without regularities in the laws of nature one would be unable to discover the laws themselves.

Hamilton's action principle. In classical mechanics the consequences of continuous symmetries are most evident using Hamilton's action principle, which postulates that classical motion is determined by an extremum principle. Thus the system is described by a generalized coordinate $x(t)$ describing its position, then the actual motion of the system, given different values of t, is such that the action $S[x(t)]$ is extremal.

The action is the local functional of $x(t)$, which can be written as an integral over time of a function of $x(t)$ and its time derivative. The classical equations of motion follow from Hamilton's principle. Symmetry of a classical system lies in a transformation of the dynamic variable that leaves the action unchanged. Thus, it follows that the classical equations of motion are invariant under the symmetry transformation. That is, since if $x(t)$ is an extremum of the action, and R generates a symmetry of the action, then $R[x(t)]$ is also an extremum. The symmetry can then be used to derive new solutions. Thus, if the laws of motion are invariant under spatial rotations, and $x(t)$ is a solution of the equations of motion, then the spatially rotated $x(t)$ is also a solution [1, p. 607].

Classical laws. After establishing this, one can better understand the more important implication of symmetry in physics in the existence of conservation laws. For every global continuous symmetry (a transformation of a physical system that acts the same way everywhere and at all times), there exists an associated time-independent quantity, which in this case is a conserved charge. Thus:

- Due to the invariance of the laws of physics under spatial transformations momentum is conserved;
- Due to the time translational invariance energy is conserved;
- Due to the invariance in change of phase of the wave functions of charged particles electric charge is conserved.

It is imperative that the (group of) symmetry be continuous, which means that it must be parameterized by set of parameters that can be varied continuously, and that the symmetry transformation can be arbitrarily close to the identity transformation. The discrete symmetries of nature such as time reversal invariance or mirror reflection, do not lead to new conserved quantities [1, p. 610].

Spatial invariance. One can give a simple geometrical argument that illustrates the connection between symmetry and conservation laws. Consider the motion $x_i \to x_f$ of a particle described by $x(t)$, from x_i to x_f. Assume that the action is invariant under spatial translations. If this is the case, then the action for the actual path, $S[x(t)]$ will be equal to the action for the displaced path, that is,

$$S[x(t)] = S[x(t) + a].$$

Consider the path of motion $x_i \to x_i + a \to x_f$. If a is infinitesimally small, then according to Hamilton's principle, the action along this path is the same as the original action, so that the difference of the two disappears. Since the action is additive, the result is

$$S[x_i \to x_i + a \to x_f + a \to x_f] - S[x_i \to x_i + a] - S[x_f + a \to x_f] = 0.$$

The action along the infinitesimal path $x_i \to x_i + a$ must be proportional to a, that is,

$$S[x_i \to x_i + a] = p * a$$

which defines the momentum p. Similarly

$$S[x_f + a \to x_f] = -p * a$$

where the minus sign is present because the path runs in the opposite direction. Consequently the momentum p is conserved. Ultimately, it is a time independent constant along the path of motion [1].

Symmetry breaking. Mechanisms of symmetry breaking are also significant in determining conservation laws. There are several mechanisms in which the symmetry of physical phenomena can be hidden or broken. The first is explicit symmetry breaking where the dynamics is only approximately symmetric, but the magnitude of the symmetry breaking forces is small, so much so that one can treat the symmetry breaking as a small correction. Such approximate symmetries lead to approximate conservation laws. Many of the symmetries observed in nature are of this type and are not necessarily symmetries of the laws of physics, but are approximate symmetries for a certain type of phenomena.

An example of an approximate symmetry is the isotopic symmetry of the nuclear force; which exists because of the small values of the fluctuations in quark masses and the weakness of the electromagnetic force. A much deeper way of hiding symmetry is called *spontaneous symmetry breaking*. Under this paradigm, the laws of physics are symmetric but the system is not, which is a common situation in classical physics. The earth's orbit is an example of a solution to Newton's equations that is not rotationally invariant, although the equations are. Consequently, for the observer of a solar system, the rotational invariance of the law of gravitation is not manifest. The particular orbit is picked out by the asymmetric initial conditions of the planet. Thus this mechanism of hiding symmetries of physics is related to the asymmetry induced by asymmetric initial values.

Rotational invariance. In mathematics, a function defined on an inner space is said to have rotational invariance if its value does not change when arbitrary rotations are applied to its argument. For example, the function

$$f(x, y) = x^2 + y^2$$

is invariant under rotations of the plane around the origin. For a function from a space X to itself (or for an operator that acts on such functions), rotational invariance may also mean that the function or operator commutes with rotations of X. An example of this is seen when the two-dimensional Laplace operator

$$\Delta f = \partial_{xx} f + \partial_{yy} f$$

is applied to a function $f(p)$ subject to rotation $r(p)$ of its argument. Then

$$\Delta f(r(p)) = r\Delta f(p),$$

or $\Delta f(r) = r\Delta f$.

Summary. Symmetry plays a fundamental role in establishing the conservation laws of physics. The main source of this symmetry is derived from Hamilton's principle, which combines Lagrangian and Laplacian concepts to basic conservation laws, as seen in angular momentum and fundamental velocity functions.

3. The Wheatstone bridge electric circuit

Contributed by Ilya Mohedano. In 1843 the English physicist, Sir Charles Wheatstone (1802-1875), found a bridge circuit for measuring electrical resistance. Although the network is named for Charles Wheatstone, who was the first to publish the network topology in 1843, the network design was apparently the work of Samuel Christie some ten years earlier. It is used to measure an unknown electrical resistance (R_x) by balancing two legs of a bridge circuit, one leg of which includes the unknown component. The Wheatstone bridge circuit, shown in Figure 3.1, has been widely used for many purposes.

Basic equations. Its equations are simple:

$$i_1 = \frac{V}{R_1 + R_2}, \quad i_2 = \frac{V}{R_3 + R_x}.$$

At equilibrium we must have $i_1 R_2 = i_2 R_x$, from which, substituting the currents, we obtain $R_x/R_2 = R_3/R_1$.

R_x is the unknown resistance to be measured, whereas R_1, R_2, R_3 are resistors of known resistance and the resistance of R_2 is adjustable. If the ratio of the two resistances in the known leg (R_2/R_1) is equal to the ratio of the two in the unknown leg (R_x/R_3), then the voltage between the two midpoints (B and D) will be zero and no current will flow through the galvanometer V_g. The resistor R_2 is varied until this condition is reached. The direction of the current indicates whether R_2 is too high or too low. If all four resistor values and the supply voltage (V_s) are known, we can find the voltage across the bridge V_g, which is given by (writing $R_4 = R_x$),

$$V_g = (\frac{R_4}{R_3 + R_4} - \frac{R_2}{R_1 + R_2})V_s.$$

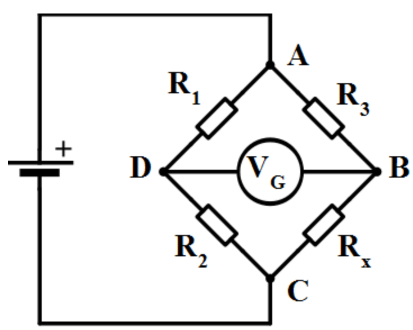

Figure 3.1. The Wheatstone bridge circuit

It further simplifies to

$$V_g = \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} V_s.$$

Symmetries in a Wheatstone Bridge circuit. In this section we study how V_g is affected when the resistors in the circuit are permuted according to certain group of permutations.

The K_4 symmetries. We have, in cyclic notation,

$$K_4 = \{1, (12)(34), (13)(24), (14)(23)\}.$$

Applying these permutations on the indices of the resistors, we obtain, respectively,

$$V_g = [1, -1, -1, 1]V_s,$$

where the coefficients (± 1) are indexed by the permutations. If we indicate the permutations by τ and the association by $\chi(\tau)$ then it follows that

$$V_g(\tau) = \chi(\tau)V_s, \quad \tau \in K_4.$$

Moreover, from the multiplication table

$*$	1	v	h	o
1	1	v	h	o
v	v	1	o	h
h	h	o	1	v
o	o	h	v	1

of $K_4 \equiv \{1, v, h, o\}$, respectively, it can be verified that

$$\chi(\tau\sigma) = \chi(\tau)\chi(\sigma)$$

for all τ, σ in K_4. That is, χ is a representation of K_4. In fact, χ is one of its irreducible characters. We may then say that the circuit's behavior is determined by one of the irreducible characters of K_4.

The D_4 symmetries. We have, in cyclic notation,

$$D_4 = \{1, (1234), (13)(24), (1432), (12)(34), (13), (14)(23), (24)\}.$$

Applying these permutations on the indices of the resistors, we obtain, respectively,

$$V_g = \left[1, \frac{r_2\, r_4 - r_1\, r_3}{(r_2 + r_3)\,(r_4 + r_1)}, 1, \frac{r_2\, r_4 - r_1\, r_3}{(r_2 + r_3)\,(r_4 + r_1)}, -1, \frac{r_1\, r_3 - r_2\, r_4}{(r_2 + r_3)\,(r_4 + r_1)}, -1, \frac{r_1\, r_3 - r_2\, r_4}{(r_2 + r_3)\,(r_4 + r_1)}\right] V_s.$$

If, in addition, we set $r_4 = r_2$, then,

$$V_g = [1, -1, 1, -1, -1, 1, -1, 1]\, V_s.$$

Similarly to K_4, and indicating the new association with the same notation, it follows that

$$V_g(\tau) = \chi(\tau) V_s, \quad \tau \in D_4,$$

and χ is one of the irreducible representations of D_4. The same result applies to permuting the resistors according to the symmetries restricted to the cyclic group C_4 (under $r_2 = r_4$).

Applications and interpretations. The original motivation for the Wheatstone network was the precise measurement of resistances. However, Wheatstone networks have arisen as structures of interest in other network situations such as traffic, pipes, and computer networks. Much of the attention paid to Wheatstone structures has centered around the networks seemingly paradoxical behavior.

Under certain conditions, connecting a Wheatstone bridge to a formerly parallel network can actually increase the total user cost of the network. First studied by Braess in the context of traffic networks, this behavior has come to be known as Braess' Paradox. The meaning of the "user cost of the network has various interpretations depending on the network of interest.

In Braess' original example the user cost of highways is the time it takes motorists to reach their final destination. An increase in the user cost, therefore, corresponds to wasted time and irritation sitting in larger traffic jams. In circuits, "user cost can be interpreted as the voltage drop across the circuit as a whole. Addition of a Wheatstone bridge lowers the voltage drop across the network (assuming the network is unbalanced to begin with); thus the "cost incurred by the Wheatstone bridge is reduced voltage over the circuit as a whole.

Braess' Paradox suggests that user costs may increase for reasons independent of the amount of traffic on the network. It has been proven that Braess' Paradox is unique to the Wheatstone network and the paradoxical behavior cannot occur outside the Wheatstone structure.

Another application of the Wheatstone bridge is low temperature alarm, as illustrated in Figure 3.2, from `http://www.wisc-online.com/`. A sensor, called "thermistor, is used in place of one of the bridge resistors. The resistance value of a thermistor varies as the temperature to which it is exposed changes. The thermistor used in this circuit has a negative temperature coefficient. As the temperature increases, its resistance decreases. As the temperature decreases, its resistance increases. At x degrees the resistance of the thermistor, along with predetermined resistances of the three fixed resistors, is at the values that make the bridge balanced. A bridge is balanced when the resistance ratio between the series resistors at the left

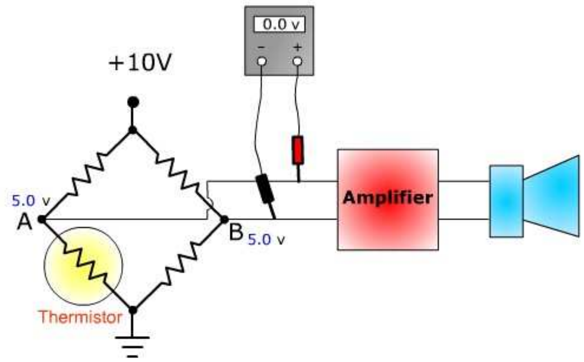

Figure 3.2. Low temperature alarm, from *http://www.wisc-online.com*.

and right branches are equal. This condition causes the bridge output voltage to be zero. Let's say the temperature drops from x to y degrees. As it does, the thermistor resistance changes. When the voltage at Point A increases to z volts more positive that the potential at Terminal B, the voltage difference between them causes an alarm to activate.

Summary. This paper has given the basic definition of the Wheatstone bridge circuit and described a modification of the Wheatstone bridge circuit in respect to the symmetry rules. We took a look at the Braess' Paradox in traffic networks which occurs only by adding Wheatstone bridge into the networks. The Wheatstone bridge has a lot of application such as traffic, pipes, and computer networks. As an example we looked at the implementation of the Wheatstone bridge as low temperature alarm. At the end, I want to note that the primary motivation for installing the Wheatstone bridge is that it may provide a reliability benefit. This reliability, however, comes at the cost of increased congestion (as in a example of the traffic). The amount of congestion actually caused is representative of the system's willingness-to-pay for flow control devices (relays, FACTS, and so on).

4. The golden ratio

Contributed by Lily Simmons. There has always been great interest in bridging the gap between the brain and consciousness with empirical evidence; a space normally only occupied by philosophical debate. A clear mind/body distinction would be a perception made by our body versus an interpretation of the perception by our consciousness. The body is the tool by which we perceive and sense; stoic and objective.

The mind, however, is the tool by which we understand and reason; lucid and subjective. Bodily perception of an object can be measured in electrical brain, but how does one measure our emotional response towards a beautiful object? This is the gap between brain and mind, and the mystery of *aesthetics*: the study of the mind and emotions in relation to the sense of beauty. There is suggestion that a mathematical proportion may bridge this mind/body by providing empirical evidence to measure beauty. This mathematical proportion is known as the known as the golden ratio. This ratio has been claimed as an aesthetical ideal in art and architecture [17], because it is the most aesthetically pleasing point at which to divide a line [18].

This ratio is not only found in man-made structures, but in the fabric of nature itself. There is an undeniable pattern throughout nature based on this mathematical proportion, but is there enough evidence to claim it as aesthetically pleasing? This brief overview of the literature looks to answer this question.

How is the golden ratio defined and constructed? A line divided by the golden ratio is defined as the ratio of the shorter segment to the longer segment is analogous to the ratio of the longer segment to the whole line. If a represents the length of the shorter segment and b represents the length of the longer segment, then the total length $a + b$ is to the longer segment a as a is to the shorter segment b, that is,

$$\frac{b}{a+b} = \frac{a}{b}.$$

Solving the quadratic equation gives the golden ratio

$$\phi = \frac{b}{a} = \frac{1+\sqrt{5}}{2} \simeq 1.618.$$

See Figure 4.1. Adding one to this number gives 1.168 and is denoted by the Greek letter ϕ. Other shapes can be derived from this ratio including rectangles, pentagons, and triangles. The process of deriving all three shapes are explained in Figures 4.2, 4.3 and 4.4, from [18].

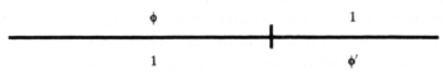

FIGURE 4.1. A line divided in the golden section. If the short segment is taken as 1, the long segment is ϕ in length. If the long segment is taken as 1, the short segment is $\phi' = 1/\phi = \phi - 1 \simeq 0.618$ in length.

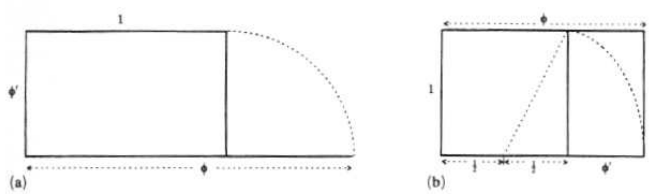

FIGURE 4.2. (a) A golden rectangle constructed by swinging vertical the short segment of a line divided in the golden section. (b) A golden rectangle constructed by swinging a line that runs from midpoint of the bottom of a square to an upper corner to a point collinear with the bottom side.

The presence of the golden ratio in nature. Examples of the golden ratio within the natural world can be seen with the Fibonacci numbers. The Fibonacci numbers are a pattern of numbers that starts with 0 and 1, and continues as the sum of the two numbers that came before the third number:

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

The limiting ratio, F, of these number approaches the golden ratio [19].

Mathematically, F and the Fibonacci sequence F_n are related by

$$F_n = F_n F + F_{n-1},$$

where F is the limiting ratio: take any number in the sequence and dividing it by the following number gives a number that approaches the limit F. The higher in the sequence, the closer it comes to F. For example, $233/144 = 1.618056$.

A pictorial representation of the sequence shown in a triangle called Mount Meru of Pingala and Pascal's triangle, as shown in Figure 4.5, from [19].

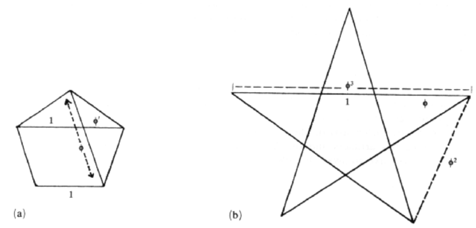

FIGURE 4.3. (a) The diagonals of regular pentagon with unit side are of length ϕ, and divide each other in the golden section. (b) A pentagram constructed from a regular pentagon with unit sides. The length of the sides of the five new triangles is ϕ; the distance between two adjacent triangle ends is ϕ^2, and the distance between the ends of two collinear triangles is ϕ^3. Each point is also a golden triangle.

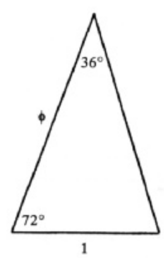

FIGURE 4.4. The golden triangle. If the base is taken as 1, its legs are ϕ in length.

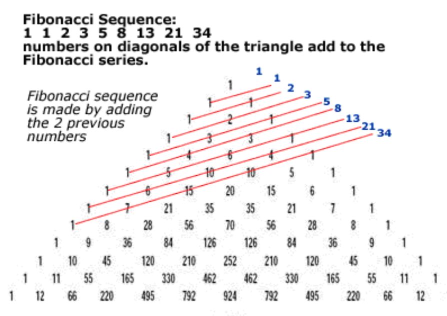

FIGURE 4.5. Fibonacci Series and Pascal's triangle. The shallow diagonals sum to the Fibonacci series, whose limiting ratio is the golden mean.

Examples of the Fibonacci sequence are present in the pattern of flower petals, leaves, and seed heads. The seeds of a sunflower are arranged in varying opposing spirals with 55 seeds going one direction and 34 seeds the other directions; both numbers of the Fibonacci sequence. It is suggested that this is the most efficient way to pack the seeds within the flower head. These spirals are not just ordinary spirals, they are known as logarithmic spirals. They can be constructed within the arcs of a golden rectangle. This is shown in Figures 4.6 and 4.7. Logarithmic spirals are found else where in nature. Various sea creatures, such as the nautilus shell, deploy this structural device [18].

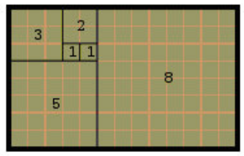

FIGURE 4.6. A golden rectangle with a tiling of squares whose sides are successive with Fibonacci numbers in length.

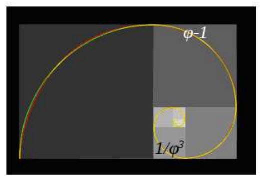

FIGURE 4.7. The golden spiral is a type of logarithmic spiral. It is made from quarter-circles tangent to the interior of each square. The length of the side of a larger square to the next smaller square is in the golden ratio.

Controversy: Bilateral symmetry vs. the golden ratio. While there is evidence to the appearance of the golden ratio in nature, is there sufficient evidence to support the claim that the golden ratio is aesthetically pleasing? This was a quest of perceptual psychologists dating back to the 19th century. A review of 130 years of literature on testing this hypothesis show inconclusive results [18]. Some studies prove the hypothesis, others show there is no preference. In [17] it is shown that there is no evidence to support the statement that the golden ratio is aesthetically pleasing.

The study [17] was conducted to see if subjects were really drawn to the golden ratio. They conducted three experiments pitting the golden ratio(1:1.618) against the unity ratio (1:1). Simple geometric figures were divided internally into these two different ratios to evaluate aesthetic appeal of both ratios. They compared a square against a rectangle whose dimensions were the golden section while also dividing each test pair internally into the golden ratio or unity ratio. All three experiments yielded results in favor of bilateral symmetry created by the unity ratio, and yielded no evidence to support that people prefer the golden ratio. They concluded it was symmetry (not the golden ratio) that is the principal determinant for aesthetic preference, and further equated it as a reflection of perceptional organizational processes instead of a unique mathematical proportion [17].

Summary. The golden ratio shows prevalence in nature but aesthetics another story. There are 130 years of literature showing studies that provide inclusive results for the relation of the golden ratio to aesthetics. Perhaps this is because most published studies only looked at shapes and lines. It is obviously easier to identify and replicate bilateral symmetry when studying lines and shapes; it is much harder to identify ratios. Perhaps the golden ratio is more intuitive. We are clearly drawn to bilateral symmetry, because it is easy to identify. Even with inconclusive evidence for the golden ratio and aesthetics, the golden ratio is present in nature. While bilateral symmetry may be the first identifiable component in the aesthetics of an image, the golden ratio provides seems to provide a harmony to compliment the image; the same way a harmony complements a melody in music. We may not notice the harmony, but it does make the melody more beautiful. This could be the reason why flowers can be so strikingly beautiful: their petals are arranged in logarithmic spirals. Whether or not there is sufficient empirical evidence aesthetic preference for lines and shapes, it can still be noted we are drawn to figures that incorporate the golden ratio.

5. Embryonic development

Contributed by Ji-in Choi. Following This section is a brief introduction to the process of “symmetry breaking observed in internal human (and other mammals) body during the embryonic development.

A human body can be considered as symmetrical when we look at it from the outside. However, most of the visceral organs are located in an asymmetrical fashion. In a normal human body, the heart is on the left side with stomach, whereas the liver is located on the right side of the body. This arrangement is called *situs solitus*. Some people are born with the less frequent organization, *situs inversus*, in which the locations of these organs are reversed. Another asymmetry is handedness. The mechanism that decides right or left-handedness of a person is cerebral lateralization.

Asymmetry in visceral organ is due to “symmetry breaking process caused by nodal ciliary rotation of an embryo. Primary ciliary dyskinesia (PCD) involves defect in the action of the cilia lining the respiratory tract and fallopian tube. Fifty percent of PCD patients are situs inversus. Many studies have related ciliary rotation and handedness [13].

Models on visceral organ and handedness orientations. . There are three models proposed in [13] to explain the relationships. The first model shown in a) and b) of Figure A.1, in the Appendix, suggests that the ciliary rotation would induce situs solitus, and situs solitus would induce right-handedness. If the ciliary rotation is absent, embryo has fifty percent chance of becoming situs solitus and right-handed, and fifty percent chance of becoming situs inversus and left-handed [13].

The second model shown in c) and d) of Figure A.1 suggested that visceral organ orientation and handedness are controlled by two independent ciliary rotations. The ciliary rotation would control orientation of visceral organs as the same manner as the first model.

The third model shown in e) and f) of Figure A.1 suggested that unknown mechanism would control handedness prior to ciliary rotation, and the ciliary rotation would control orientation of visceral organs as the same manner as other two models.

Application of symmetry in orientation pairs. According to three models represented in Figure A.1, an embryo has multiple choices of path that can take in regards to orientation of visceral organs and handedness in each model. The results of the paths are shown on Figure 5.1.

For convenience, the organ orientation on the left was labeled as L, on the right as R, left-handedness as L and right-handedness as R. Four different symmetric patterns were observed, and they were represented in Figure 5.2. Since the orientation of liver is always opposite, and the orientation of stomach is always the same as heart, orientation of one organ can represent orientation of other organs as well. In this study, we used the orientation of heart.

According to the first model, A) and D) from Figure A.1 are observed. According to the second model, all four patterns, A), B), C), and D), are observed. As in second model, the third model also explains all four patterns. Let’s say we label each end point of each “x” as 1,2,3 and 4 starting from the upper left and moving clockwise (upper left point as 1, upper right as 2, lower right as 3, and lower left as 4).

Pattern A) has a symmetry of an identity (itself), a symmetry of vertical reflection (pattern D), and a symmetry of 90 deg rotation (pattern D) within the set of possible orientation pairs. Similarly, pattern D) has a symmetry of an identity (itself), a symmetry of vertical reflection (pattern A), and a symmetry of 270 deg rotation (pattern A). Pattern B) has a symmetry of an identity (itself) and a symmetry of vertical reflection (pattern C). And pattern C) has a symmetry of an identity (itself) and a symmetry of vertical reflection (pattern B).

Summary. It is ironic how symmetry is observed in the biological process of symmetry breaking during the embryonic development. Compared to the symmetries of the square, less number of symmetries were observed in four patterns of organ/handedness orientation pairs. This is a consequence of the fact that the

model	path # within the model	Pair result from each path	orientation of organ or handedness			
			heart	liver	stomach	handedness
Model 1	1	1, A (solitus, right)	L	R	L	R
	2	1, A (solitus, right)	L	R	L	R
	3	2, B (inversus, left)	R	L	R	L
Model 2	1	1, A (solitus, right)	L	R	L	R
	2	1, A (solitus, right)	L	R	L	R
	3	1, B (solitus, left)	L	R	L	L
	4	1, A (solitus, right)	L	R	L	R
	5	1, A (solitus, right)	L	R	L	R
	6	1, B (solitus, left)	L	R	L	L
	7	2, A (inversus, right)	R	L	R	R
	8	2, A (inversus, right)	R	L	R	R
	9	2, B (inversus, left)	R	L	R	L
Model 3	1	1, A (solitus, right)	L	R	L	R
	2	1, A (solitus, right)	L	R	L	R
	3	2, A (inversus, right)	R	L	R	R
	4	1, B (solitus, left)	L	R	L	L
	5	1, B (solitus, left)	L	R	L	L
	6	2, B (inversus, left)	R	L	R	L

FIGURE 5.1. Orientation of visceral organs and handedness.

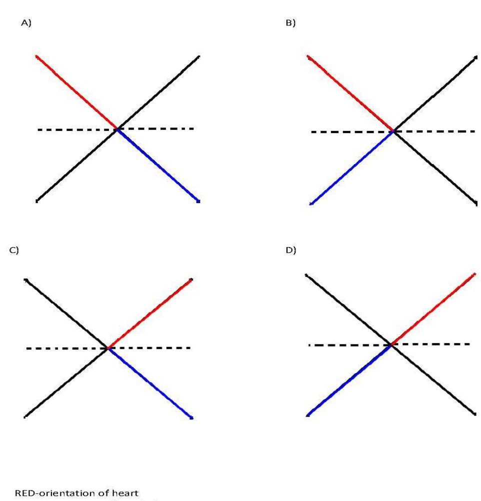

FIGURE 5.2. Representation of four possible orientation pair patterns.

orientation of internal organs and handedness are distinct properties, and therefore cannot be interchanged, unlike the identical sides of a square.

6. The writings of J.L. Borges

Contributed by Katie Kemp. There are many characteristics which distinguish one author from all the others. For the well-known Latin American writer Jorge Luis Borges, his characteristic use of the concepts of mirrors, language, entanglement, and the afterlife establish his style as mystical and above reality. Within his use of these concepts, Borges also simultaneously reveals an aspect of symmetry regarding his method of half-hiding and half-revealing his message, entangling consciousness and reality, and ultimately leading his readers onto an unending process of words intertwined with language which creates an ambiguous understanding for the reader of what reality really is.

In Borges' poetry there are various images that he uses and repeats in order to create an idea that is half-exposed, but also half-disguised. For example, in his poem *We Are The Time. We Are The Famous* he states,

> We are the river and we are that Greek
> that looks himself into the river. His reflection
> changes into the waters of the changing mirror,
> into the crystal that changes like the fire.

In this excerpt, Borges describes a reflection using his common image of the mirror as a means of partially revealing his message (which in this case deals with the relationship between reality and our perception of it that which is articulated with the mirror), but not fully unveiling his intention. With a mirror we can see the outside appearance of an image but not the actual image, therefore it is an allusion of reality and therefore only showing us part of what is really there.

In addition to escaping reality, Borges also entangles language by making the readers understanding half-concealed and half-exposed by having his protagonists go on journeys that create expectations in which the reader believes certain happenings are certain, but in reality end up in other ways. This use of certainty and uncertainty creates ambiguity for the reader and thus thwarts the ultimate meaning. According to [2], "the labyrinth comes to represent the ultimate meanings-of life, scholarship, and historyand is finally transformed into an image of the symbolic workings and power of language itself." Hence, by incorporating his specific style of language and ambiguity, Borges' message is only half-hidden, therefore only half-understood, thus creating symmetry. With the cyclical use of mirrors and his methods regarding the delivery of his message, Borges simultaneously divulges his theme of the entanglement of our consciousness and our perception of reality.

With this entanglement of consciousness and reality, Borges utilizes an unending process of words and language in order to create this illusion of reality while simultaneously bringing our consciousness into another realm. In his works, Borges often puts the reader into another world allowing them a glimpse into their human consciousness while also allowing the reader to escape reality. In his poem, *To A Cat*,

> Mirrors are not more silent
> nor the creeping dawn more secretive;
> in the moonlight, you are that panther
> we catch sight of from afar.
> By the inexplicable workings of a divine law,
> we look for you in vain;
> More remote, even, than the Ganges or the setting sun,
> yours is the solitude, yours the secret.
> Your haunch allows the lingering
> caress of my hand. You have accepted,
> since that long forgotten past,
> the love of the distrustful hand.
> You belong to another time. You are lord
> of a place bounded like a dream.

In this poem, Borges combines our reality (Ganges, moon, and the sun) with that of another realm (dreams, divine law, and afar). The mixture of these two concepts creates a parallelism and a repetitious tradition in which Borges combines our reality or that of which we can understand, with our consciousness, to that of which we can imagine or perceive.

Summary. In conclusion, the symmetry found in Borges poetry is utilized not only to bring a sense of non-reality to his readers, but also to allow us to see beyond our consciousness as humans. Borges uses the image of the mirror and the concept of mixing words and language in order to create his magically symmetrical style. His common and traditional use of these methods represents a cyclical presentation of his poetry that he is famous for.

7. The drawings of M.C. Escher

Contributed by Rebecca Ranay. M.C. Escher's impressive artistry has been admired by fans for many years. His unique creativity sets him apart from his peers despite the simple concepts of symmetry that can be followed by many. His pieces all relate to the various techniques involved in creating and describing symmetry.

Periodicity. A common characteristic found in Escher's work includes his patterns being periodic. This term describes a single symmetry group within a pattern that translates itself in linearly separate directions. Only 17 groups of periodic patterns exist, they are called the plane symmetry groups [3].

Plane Symmetry Groups. Of the 17 plane symmetry groups, Escher focused mainly on the 7 types that exclude reflections [3]. These individual groups are named p1, p2, pg, pgg, p4, p3, and p6 [4].

Symmetry group p1 shows to be one of the simplest groups. In Escher's drawing no. 105, showing in Figure 7.1, he uses a parallelogram lattice for the generating tile and translates it continually about all edges of the tile. Take note that the coloring of the pattern can be ignored in order to obtain a smaller starting tile. The p2 group can be seen in Escher's drawing no. 88, showing in Figure 7.2, completely tiled by sea horses.

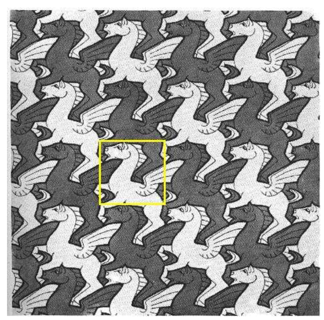

Figure 7.1. Escher's drawing no. 105

Like p1, a parallelogram type lattice is used for the original tile and neither reflections nor glide-reflections are used. However, unlike p1, p2 rotates by 180 degrees.

Escher represents the symmetry group pg in his drawing no. 63, shown in Figure 7.3. A rectangular lattice can be drawn over a specific area of the pattern and used as the standard tile. This piece is then repeated using the glide reflection technique. The complete pattern is composed of the same tile that is constantly translated horizontally and reflected about the vertical axis.

Escher's lion drawing, Figure 7.4, reveals the use of the pgg symmetry group. His design illustrates having only two-fold rotations and glide reflections, characteristics of pgg [4].

In his lizard tessellation no. 104, shown in Figure 7.5, Escher employs the p4 symmetry group. In this, the tile is rotated about its corner edge at two angles: 90 and 180 degrees. The larger square that encompasses the four rotations of the prototype tile is then translated both horizontally and vertically .

The last two of the symmetry groups that Escher utilizes in his drawings are p3 and p6. Both are similar in that they have a hexagonal lattice and neither contain reflections nor glide reflections. The difference is that p3's and p6's highest order of rotation is 3 and 6, respectively. This meaning that the prototype tile of p3 rotates 120 degrees about a corner and that of p6 rotates 60 degrees.

FIGURE 7.2. Escher's drawing no. 88

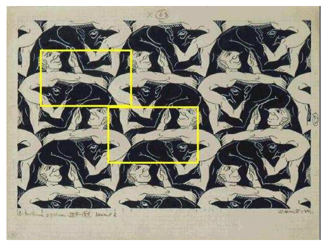

FIGURE 7.3. Escher's drawing no. 63

FIGURE 7.4. Escher's tessellation with lions (pgg group)

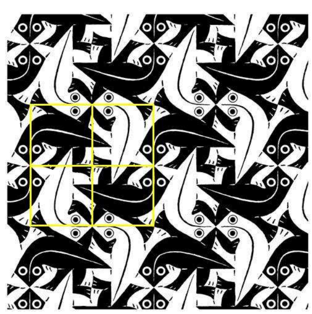

FIGURE 7.5. Escher's drawing no. 104

Motif Coloring. The aforementioned characteristics refer to the entire layout of the pattern whereas the specific coloring of the motif distinguishes each sub-pattern from one another. Escher employs the map-coloring principle in which two adjacent copies of a motif will be differently colored [3]. Imagining a checkerboard pattern will help to better understand this concept. Each square alternates black and white, no two squares that share an edge are of the same color; that is, no two black squares will touch and no two white squares will touch each other. This pattern can then be further classified as a 2-colored pattern and as perfectly colored. The "2-colored" pattern name corresponds to the number of colors that every motif copy receives. Thus, a pattern with n colors is "n-colored". The term "perfectly colored" describes when the permutations in the symmetry create a subgroup of the larger permutation group (S_n) with n colors [3].

Summary. Most of Escher's work involves 3 or 4-colored patterns of symmetry. Additionally, except for two of his patterns, Escher has perfectly colored symmetry. After several years of creating patterns and the influence of mathematicians such as George Polya, Escher began to develop his own system in designing motifs, mainly easily identifiable animal motifs [3].

8. The Old Testament Trinity

Contributed by Opal Boonpitak. Andrei Rublev was a medieval Russian monk who was and still is well-praised for his intricate work in the awe-inspiring icon *The Old Testament Trinity.* Alexander V. Voloshinov, a mathematician and philosopher from Saratov State Technical University, philosophically and theologically devised a few symmetry themes based on his observations on Rublev's celebrated piece of Russian art. After having undergone such processes, Voloshinov not only is able to support the impact of symmetry in a secular sense, but, also, in a spiritual manner. Following [8], we will explore two of Voloshinov's symmetry themes in Rublev's famous Old Testament Trinity, shown in Figure 8.1. These two themes are *The Three Circles of the Trinity* and *Revelation: Mirror Symmetry.* We will also analyze the relation of the icon's meaning to its biblical story and the colors of the icon.

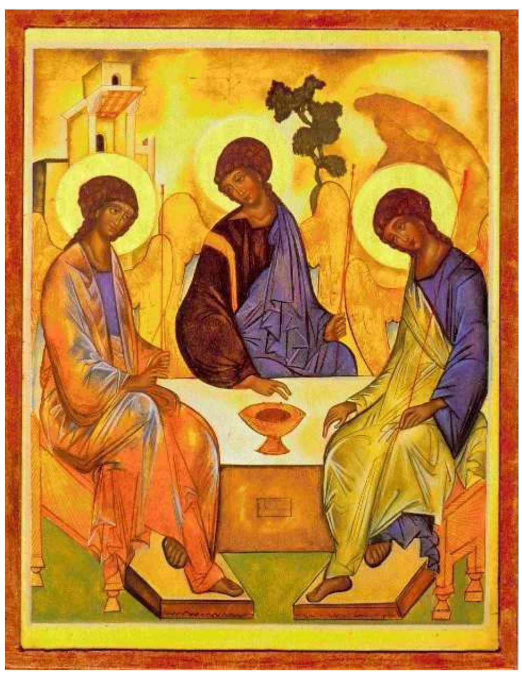

Figure 8.1. The Old Testament Trinity. Andrei Rublev (Ca.1410-20) Tretiakov Gallery, Moscow, 142 x 114 cm.

The Three Circles Of The Trinity. Voloshinov looked into three circles that were constructed on the icon in a particular symmetric order by many art critics. The reasoning for the seeking of circles within this piece is because, prehistorically, the circle has been considered the most perfect shape;

of all two-dimensional figures only a circle coincides with itself at any turn round its center, possessing the highest degree of central symmetry. Because of this, from ancient times different cultures have used the circle to symbolize the sky, the path of the sun, the sphere of the heavens-everything lofty, perfect, eternal and close to God.

Figure 8.2 represents Voloshinovs research on the placement of the three circles by Dante according to a golden property of a rectangle:

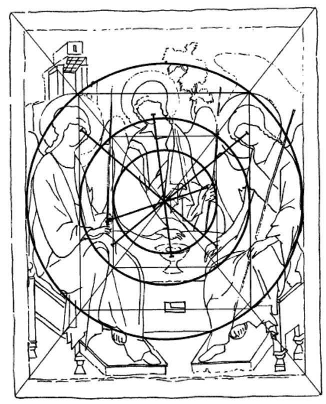

FIGURE 8.2. Three "golden circles" of the Trinity, from [8].

The three circles naturally lie on significant elements of this icon. There is the circle of the three faces, the circle of the arms and wings of the angels on the sides, and the circle of the arms of the angel in the middle and the cup. There have been debates on whether the middle angel is God the Son, whom is put higher than God the Father and goes according to the Orthodox church, or the three angels are equal, which goes according to the Catholic church. However, Rublev established his icon in a manner that all are equal in the circle.

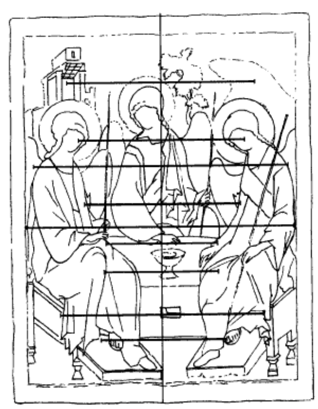

FIGURE 8.3. The Revelation: mirror symmetry, from [8].

Revelation: mirror symmetry. The type of mirror symmetry that this icon holds is the vertical axis or the plane of symmetry (bilateral symmetry), where in nature, this type of mirror symmetry portrays the stability and equilibrium of an object.

Regarding Figure 8.3, the left and the right sides of this icon are mirror-symmetrical. Looking at the angels on the left and right side, we see how the outline of their backs correspond exactly. Also, the centers of their faces correspond exactly with slight shifts of their halos. The positions of the feet and the pedestals are slightly altered. In the background, the temple or house on the left corresponds to the mountain, or hill-like figure to the right. The mirror symmetry is broken in the outline of the central angel.

This asymmetry is rather exciting, though, because it is the source of movement and energy in this icon. Voloshinov interestingly enough philosophizes on how symmetry and asymmetry inter-wove to portray some truth in the Trinity; "the asymmetry of the center exploded in the center and calmed down in the mirror symmetry of the right and left sides". From these findings, Voloshinov hypothesizes that the angel in the middle is God the Father.

Illumination: The three cups of The Trinity. The beauty of this icon continues with the detection of the shapes of three cups: the cup on the Communion table, the cup made up by the inner outlines of the angels, and the cup made up by the outer contours of the angels and thrones [8]. The first cup, the inner cup, is Abrahams sacrifice for Faith [8]. Although The Trinity emphasizes on the three angles, this icon was based on the Old Testament tradition of Abraham and his wife Sarah good-naturedly meeting three angles under the shade of the Mamrean Oak. Abraham had told his wife to prepare a fine meal for the angles (three measures of fine meal [8]) and sacrifice a lamb (dress a calf tender and good [8]). Together, Abraham and Sarah served the angels, but they inferred this service as for the Lord, Whom was in their home. They had Faith in the Lord God and his divination that Sarah would give birth to a son [8].

The second cup is the representation of God the Fathers sacrifice for the Love of Humanity [8]. Analyzing the icon, if we interpret the middle angel as Jesus Christ, the Son of God went according to his Fathers will to be the Savior of the world and Man. Jesus Christ died for human sins with his own sufferings [8]. From the first cup, we interpret that the Father sacrificed his Son, to the second cup in which His Son sacrifices himself for the love of humanity [8].

The third and final cup, the outer cup, signifies the Trinitys sacrifice for the Hope of Salvation [8]. This third cup is made up by the Trinity and contains the whole Trinity, for God the Father and the Holy Spirit cannot leave His Son alone in the second sacrificial cup because the Trinity is undividable [8]. Therefore, God the Father and Son of God make up the third and greatest sacrificial cup, for which it includes themselves. the sacrifice for Love will not be in vain, and the whole Trinity sacrifices itself for the Hope of Salvation [8].

Colors and their interpretations. Rublev enhances our visual senses for better interpretation of this icon by using such appropriate and true colors for the garments clothed on the angels. The angel on the right represents the Holy Spirit. The garment of the Holy Spirit contains a blue color that can signify the sky. The garment is wrapped over with a green robe. These colors can demonstrate that the Holy Spirit flows in sky and water and that it breathes in the heavens and the earth. The touch of the Holy Spirit brings all things dead to life.

The angel in the middle represents the Son of God, whom has richly deeper colors than the other angels. The garment is of reddish-brown color, which symbolizes the earth, along with a blue cloak, which can signify the blues of heaven. These colors may demonstrate the unification of the heavens and the earth, for the Son of God is The Father sent from the heavens to the earth as Jesus Christ in flesh; the natures of heaven and earth are present in Christ. The settle gold over his right shoulder is a gold band that obstructs his earthly garment, for it represents the holiness that fills the flesh-being of Christ.

The angel on the left represents God The Father, whom seems to be wearing all colors in a sort of fabric that changes with the light. The garment may be considered transparent, which cannot be fathomed, for God cannot be described because no one has seen The Father. He leaves us in awe. He is. the vision of him fills the universe. These interpretations were abstracted from `http://tars.rollins.edu/Foreign_Lang/Russian/ruspaint.html`

Summary. Andrei Rublev's astonishing work in the icon The Old Testament Trinity leaves viewers of his masterpiece in awe. There is so much depth to it, and when we realize how significant of a role symmetry played in this icon, not only can its role explain its secular impact on us in which it is pleasing to the eye, but also its spiritual impact in which it goes above and beyond our eyes.

9. Dancing

Contributed by Monica Wojciechowski. Dance is an adventure of the body through space of infinite movements [5]. These movements that comprise this adventure are intertwined with symmetry and create a vivid performance for an audience. To create a dance routine, dancers select movements that may portray emotions, symbolize a story, or contain no specific meaning just when combined they appeal to the audience visually [6]. Therefore, dancers posses a language of movements and techniques that involve symmetry. Upon constructing a dance, the following techniques are used: fixed point technique, reversals, conversions, as well as global and local.

Fixed Point Technique. Fixed Point Technique allows the position of any part of the body to be described as related to another body part within space [5]. This relationship is intrinsic. The fixed point technique allows a dancer to create a relationship between two body parts or freeze a specific part of the body while moving through space. For example, a dancer may keep their arm in the air while in one dance position. That arm, while in space, has a relationship with the upper body that is unchanged. When the dancer changes their position, they may place their arm down, changing the relationship between the arm and upper body. The intrinsic relationship between the arm and upper body has been altered with the change of dance positions. When the dancer alternates relationships between body parts in space, they create infinite movements, which attract the eyes of an audience.

Reversals. Fixed Point Technique focuses on maintaining a specific part of the body immobile during dance. Reversals, however, focus on reversing the limb that initiated a movement. This technique requires the dancers understanding of how parts of the body collaborate to generate a movement. A simplified example of this technique includes tracing the left arm with the right hand. In order to obey the reversals technique, a dancer must then trace the right hand with the left arm. Therefore, the body part that was responsible for commencing the action was reversed. Initially, the hand was responsible for movement. Ultimately, the arm created the movement. Reversals force the audience to trace a dancer's movements because although the movements may be of similar nature, they are in fact distinct [5].

Conversions. Conversions are yet another technique incorporated in dances to stimulate attention from the audience. Conversions are defined as concurrent, repetitive, bilateral movements [5]. In other words, during this technique a dancer moves multiple parts of the body, the movements are cyclical and both sides of the body are used. This technique can be further varied by manipulating the placement of body parts along an imaginary x, y, z axis, or by changing the magnitude of the movements. All limbs are involved in the technique of conversions, which may be visualized by a dancer performing multiple turns. Not only is the dancer moving their legs in a cyclical motion, but their arms also fluctuate between being located above or below the head.

Global and local. Global and Local technique incorporates all of the previous techniques and it refers to the order and layering of movements [5]. Local movements correspond to movements that occur in a small vicinity of the body. Local movements may be as simple as the difference between a foot with pointed toes, and one with toes that are curled. Global, on the other hand, refers to movements that may involve the entire body. Global movements are pronounced. For example, when a dancer switches the dance position from a pirouette to the splits, multiple body parts are responsible for the transformation. Dances are constructed when local and global movements are combined and repeated.

Labanotation. The previous dance techniques, as well as any dance or performance, may be expressed symbolically by using Labanotation. Labanotation is a system devised to notate body movements [7]. This system was invented by Rudolf Laban, and it is used internationally as a clear and logical form of communicating body movements. Labanotation is based on a series of symbols and pictures which, when interpreted by the reader, can be recreated and performed. Labanotation reveals four aspects of body movement: the type of movement; the rhythm and duration of the movement; which part or parts of the body perform the movement; and the sequence of movements [7]. These aspects parallel the previously described techniques of

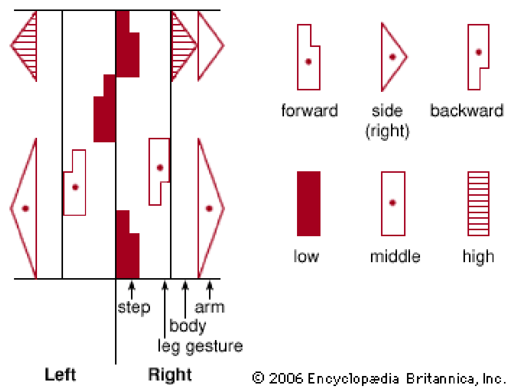

FIGURE 9.1. Elementary symbols and their meaning in Labanotation. From www.britannica.com.

dance. Labanotation is, therefore, a written language of dance. Figure 9.1 illustrates the elementary symbols and their meaning in Labanotation.

Summary. Dance cannot be defined by a single movement of a part of the body. Therefore, it is the combination of the above techniques that define a dance. These techniques, when united, allow the dancer to become the center of attention. The dancers techniques, which result in various body movements, portray symmetries that invoke interest from the audience. The movements that a dancer performs, as well as any dance, can be compiled and notated in written form, using Labanotation.

10. Sexual attractiveness

Contributed by Noa Krugliak. Early cultural practices such as body and facial painting are characterized by symmetric design (see Figure 10.1). Such practices have been shown to develop through sexual selection. This has been explained by the fact that symmetry is perceptually silent in human visual processing, it is prominent in memory, valued even in infancy, and has a positive correlation to attractiveness across cultures. These truths highlight the fact that the preference for symmetry is rooted in evolutionary history and affected by more than cultural practices alone [9].

Symmetry has been shown in many studies to have a role in the perception of attractiveness. One of such studies examined the perceptual attractiveness of symmetrical and asymmetrical faces as well as faces that were decorated with symmetrical and asymmetrical designs. It was shown that symmetrical faces are perceived as more attractive. Additionally, symmetrical facial design increased asymmetrical faces' attractiveness, while asymmetrical design decreased the attractiveness of symmetrical faces.

Many explanations and hypotheses have been proposed to explain this phenomenon. According to the good gene hypothesis, symmetry deviation from bilateral symmetry, also called fluctuation asymmetry (FA), is due to an organism's inability to cope with environmental stressors. Symmetry signals a stable development therefore, a greater level of fitness and higher mate quality [10].

Another hypothesis is called the bias hypothesis, which explains the symmetrical preference as a byproduct of the biological system's ability to identify different objects in various positions and orientations.

A third hypothesis, the extended phenotype hypothesis, explains the preference of symmetrical art for its ability to signal the fitness of the artist by attributing perfect symmetrical art work to be harder to produce [9].

Is the symmetric = attractive paradigm true? The role of bodily symmetry in perception of attractiveness has been studies by many. There are many variables that influence perception of attractiveness which might obscure the role of FA, such as waist-to-hip ratio (WHR) and body mass index (BMI). A study aimed to eliminate these factors used special software, which averaged the images of left and right sides of the body, creating an image representative of the original body but perfectly symmetrical. This morphed image, shown

FIGURE 10.1. Examples of symmetrical face painting, from [9].

in Figure 10.2, identical to the original in WHR and BMI but that differs only in its symmetry [10]. This

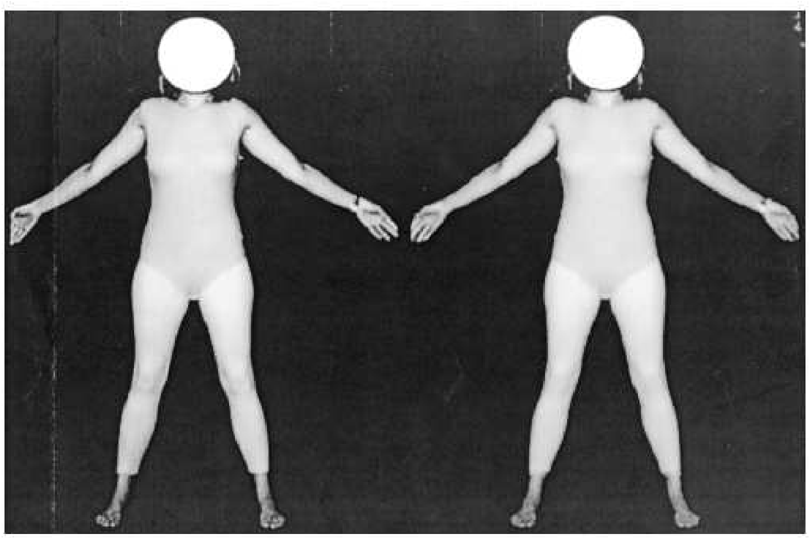

FIGURE 10.2. example of a normal (asymmetrical) image (left) and a morphed (symmetrical) version (right) [10].

study has shown no significant difference in perception of female attractiveness between the symmetrical and asymmetrical images. Symmetry played a role only when participants were forced to choose between symmetrical and asymmetrical images. It was concluded that body symmetry plays only a small role in perception of female attractiveness compared to BMI or WHR. See also Section 11.

Is FA a good indicator of fitness, fertility and health? While it was shown that symmetry (low FA) plays a role in perception of attractiveness, and many claim this phenomenon to be an evolutionary trend, the question is whether or not FA is an accurate indicator of mate fitness. It was shown that woman with higher breast-to-underbreast ratio (large breast) and woman with relatively lower WHR, characteristics considered attractive by men, were found to have higher reproductive potential, indicated by 17-b-oestradiol (E2) and progesterone (P) levels compared to women with smaller breasts and higher WHR. E2 levels correlate with follicle size. Also, P and E2 alike correlate with the quality of the oocyte and the edomertium of women[11]. Studies have shown that higher levels of E2 and P during the menstrual cycle are the ones to result in conception rather than non-conceptive cycles in the same women [12].

Summary. Symmetry was shown to play a role in perception of attractiveness. Symmetry of the body, face and facial design was found to increase attractiveness. Yet, when symmetry of the body was studied more closely, other co-factors were found to effect the perception of attractiveness (BMI and WHR). When these features were factored out, symmetry was found to have a less significant role than BMI and WHR in the perception of attractiveness. To test the biological relationship of symmetry and attractiveness, FA (symmetry) was compared with womans fertility, and the two were found to be directly related. Woman who showed higher level of symmetry were found to have higher level of the hormones estradiol and progesterone, therefore, have higher level of fertility. Rapidity human's attraction to symmetry appears to be a phenomenon rooted in man's evolutionary history of sexual selection.

11. Facial attractiveness

Contributed by Keila Valle. Many believe that humans find others attractive because of symmetrical features. This is a method often seen in animals finding mates. In art, symmetry has been looked upon as beautiful and has inspired many famous painters.

Human symmetry is often perceived along a vertical axis. While humans can perceive and be attracted to symmetric patterns, attractiveness also has to do with personality and preferences that humans have. Individuality and talents among other characteristics contribute into what a person perceives as attractiveness. This is especially true when a person is looking for a partner with whom to spend the rest of their lives.

When one is attractive, one provides pleasure in appearance or in manner. When saying that symmetry is what defines a person's attractiveness, the assumption is made that personalities and other characteristics in a person are secondary to symmetry; that is the symmetry hypothesis. What was found in two independent researches is that this hypothesis is not supported.

Research I. There were two parts in Little's [21] research. The first part consisted of conducting a poll in which participants were presented with 15 pairs of pictures of human faces, both male and female. Participants were presented with one pair of faces at a time. The pair consisted of a face and its symmetric counterpart. The first instruction given was to choose which they felt was more attractive. When they finished, the second part of the experiment was conducted. Before the second half began, they were told to choose which face was more symmetrical. The symmetry rating was always conducted after the attractiveness rating.

The results showed that "When examining individual scores, a person's preference for symmetry was not correlated with their ability to detect it." What this demonstrates is that when a subject chose a face that appeared to be more attractive, they did not consider symmetry as part of their choice. The subject's choice may have had to do with other factors such as expression, specific facial features, or first impression of the faces the subject was presented with, among others. Because the ability to detect symmetry and the preference for symmetry is not linked together, this shows that symmetry may partially affect attractiveness, but not directly.

Research II. Rhodes's [22] research also consisted of two components. The first part consisted of averaging the images of 24 faces with special morphing software. As opposed to Little's research, participants were chosen around the same age. The group consisted of 36 undergraduates (18 male and 18 female). Participants were asked to choose who they believed was more attractive. In the second experiment the averaged faces were made symmetrical. Participants were asked which faces they deemed more attractive, adding in that they should look to see who they would believe they could see as a potential life partner.

In [22], the abstract mentions, Several commentators have suggested that the attractiveness of average facial configurations could be due solely to associated changes in symmetry. If this symmetry hypothesisi s correct, then averageness should not account for significant variance in attractiveness ratings when the effect of symmetry is partialed out. Rhodes research disproves the symmetry hypothesis. As stated in [22], "Although symmetry explained significant variance in attractiveness judgments when the effects of distinctiveness (converse of averageness) and expression were taken into account, distinctiveness still accounted for significant variance in attractiveness. This suggests that even when expression is taken into account, symmetry does not remain the sole factor of attractiveness. There is something more that is sought.

When reviewing these experiments, it can be argued that humans prefer symmetry because symmetry it pleases the human eye. Also, symmetry is indicative of a healthy human body. While this may be true, it is also true that while symmetry is looked upon as attractive in some, it is not independent of cultural factors. Two people with different backgrounds may respond to others distinctly due to what their impressions are. While it may or may not be true that certain cultures show a preference for symmetry, it is not true that this is a mechanism that humans have as a whole. Having seen this through experiments establishes that there is something more contributing in what a person deems as attractive.

Summary. Symmetry may have an impact on who a person deems attractive, however, symmetry is not the only characteristic humans seek. Other factors contribute to human attraction.

12. Social relations

Contributed by Lidija Relja. Hearing sayings, such as "the apple does not fall far from the tree" or "you have rubbed off on me" portrays the idea that growing up or spending a majority of time with certain people has a great effect on one's mannerisms and behavior. A sort of symmetry can be experienced between individuals constantly in close quarters. This symmetry can be established by individuals themselves or by society.

Society, in general, creates symmetry in the specific roles individuals are expected to portray based upon his or her age, gender, and background. Symmetry is such a powerful concept that is has the ability to trigger severe health consequences if not presented correctly in social situations.

Numerous studies have been completed to test how great of an extent symmetry presents itself in social relations. Spousal enjoyment, sibling roles and relations, and the correlation between symmetrical social exchange and one's health will be discussed in greater detail in this section.

Spousal symmetry. According to [14], a predominantly emerging family entitled the "symmetrical family" is becoming ever more popular in this society. A symmetrical family is one where "husbands are more at work inside the home, wives more outside." There is a general balance between roles and responsibilities of both spouses in this type of family structure. This study went into great detail on which specific everyday activities are enjoyed by the husband and wife. It also sought to determine "how much more likely it is that a marital partner will enjoy each activity when his or her spouse also enjoys that activity." The study data shows that there is a similarity in the level of enjoyment of activities between husbands and wives; this symmetry is present in activities such as walking with others, music and art, and active sports.

For example, visiting relatives is enjoyed about twice as much by wives compared to husbands; however, the husband's enjoyment due to visiting relatives increases drastically. It is a noticeable characteristic that overall, a spouse's influence increases the overall enjoyment of a specific activity. There is no doubt that symmetry can be found between spouses in pleasurable activities. Symmetry was also accomplished to a greater extent based on the roles portrayed by husbands and wives. It was confirmed that "family oriented" husbands are more likely to have a more symmetric couple relationship compared to husbands who had a larger focus on their careers. Two factors were found to determine whether a wife will have a more symmetric relationship with her husband which include her position on having a career and her “enjoyment pattern outside the home following marriage.” If a wife believes that women should have a career outside of the home, she was termed “integrated.” Integrated wives, in fact, were found to be more likely to participate in symmetrical couple relationships.

It is increasingly more evident that the husband’s role is more pivotal, meaning his role of being family oriented has a higher effect on symmetrical relationships compared to the integrated wife persona.

Sibling symmetry. With the large amount of time siblings spend with one another, it is almost inevitable that certain traits or actions become symmetrical between individuals. Roles are also defined that provide a symmetrical relationship between siblings. In [15], roles of both younger and older siblings are defined. Older siblings tend to be more aggressive and in charge while younger siblings are more “imitative and passive.” Older siblings usually tend to act as role models or guidance counselors to those younger siblings who seek advice. These roles can be seen in numerous brother and sister relationships showing quite a correlation in the roles undertaken by siblings based on his or her age.

The study discussed in detail how family background affects siblings in his or her intellectual skill. Previous studies have confirmed what is usually assumed; parents who have obtained more education and have a higher income generally have kids with “greater intellectual ability.” The study wanted to see if intellect was symmetrical between siblings rather than different families, and included various sibling combinations based on ordinal position, or age, and sex. The results show that siblings in most cases had similar intellectual abilities regardless of age and sex. The data produced in this study shows that there is a great deal of symmetry in the level of intellect between siblings. This could be explained since siblings are exposed to the exact same family background and situations; therefore, each sibling “benefits equally from resources that parents provide for their children” despite the age old idea that parents spend more time and money on older siblings rather than younger ones.

The study strengthened the original idea stating that despite age and sex of an individual, if similar family background is provided to each sibling, symmetry and resemblance in intellectual skill will be present between siblings.

Effect on health. Following [16], symmetry has been found to play quite a role in the overall health of individuals. A career is a major component of any adult’s life for it is the basis of which one obtains money used to pay for all of life’s necessities and desires. By spending such a large deal of time in a work environment, trust must be achieved between an individual and his or her job along with other coworkers in order to provide a healthy lifestyle for the individual. Trust is such an important aspect because it not only provides a “sense of security in the work setting, but it also motivates one to participate in social exchange where losses are lower than the expected gain.

However, more frequently than not, individuals are subjected to a non-reciprocal exchange at work, where such a great effort on the part of the employee is downsized by a lack of rewards, which can include money, self-esteem, and job security. In such cases where symmetry of social exchange collapses and reciprocity has failed, an individual would try to discover a more appealing job that provides a higher reward than a loss. Unfortunately, this is not always the clear solution. Many individuals, especially those in unskilled jobs, elderly persons, or disabled adults, are forced to remain in this environment because simply, there is nowhere else to find a job and a weekly income is so much important than working in this type of setting. Little do they know that by staying in this unhealthy way of life, they are eliciting negative emotions, such as anger and stress, which contribute to health problems such as coronary heart disease and depression.

In [16] it is stated that one will not leave an asymmetrical work setting due to dependency, strategic choice, and over commitment. Individuals become dependent on a job because it provides an income for daily expenses. If one leaves a career, the income is lost as well and more problems immediately arise. One may see a job as a strategic choice, meaning that he or she believes a promotion can be achieved by obtaining job experience in the beginning. This is most common in jobs with heavy competition. Over commitment is a quality used to describe an individual who gives nothing less than 200% at all times. Quitting an asymmetrical social exchange job would signify giving up and that is not acceptable to over-committed individuals.

If one continues to remain in an environment where an imbalance is present between the hard effort one puts in and the low rewards one receives, he or she becomes more at risk for stress-related disorders. It was found that people who had encountered "failed reciprocity while at work were two time more likely to suffer from health disorders, most commonly coronary heart disease. There was also evidence supporting that it is more likely to occur for men, more specifically middle-aged men, than for women. Breaking the symmetry of social exchange can be so detrimental to ones health that it should be considered carefully when evaluating a work situation; if not, the sacrifice will be much greater than the reward if health problems result as a consequence.

Summary. The above described studies all show symmetrical relationships and traits are present between spouses and siblings. It also considers how strongly related a symmetrical work environment and health really are. This is all taken as a general case; there are always occurrences that differ from the results obtained in each study. Overall, it is common for individuals in close relationships, with either other individuals or work environments, to show similarity of actions, thoughts, personalities, and feelings. In fact, the apple might fall much closer to the tree than one had originally thought.

Appendix A

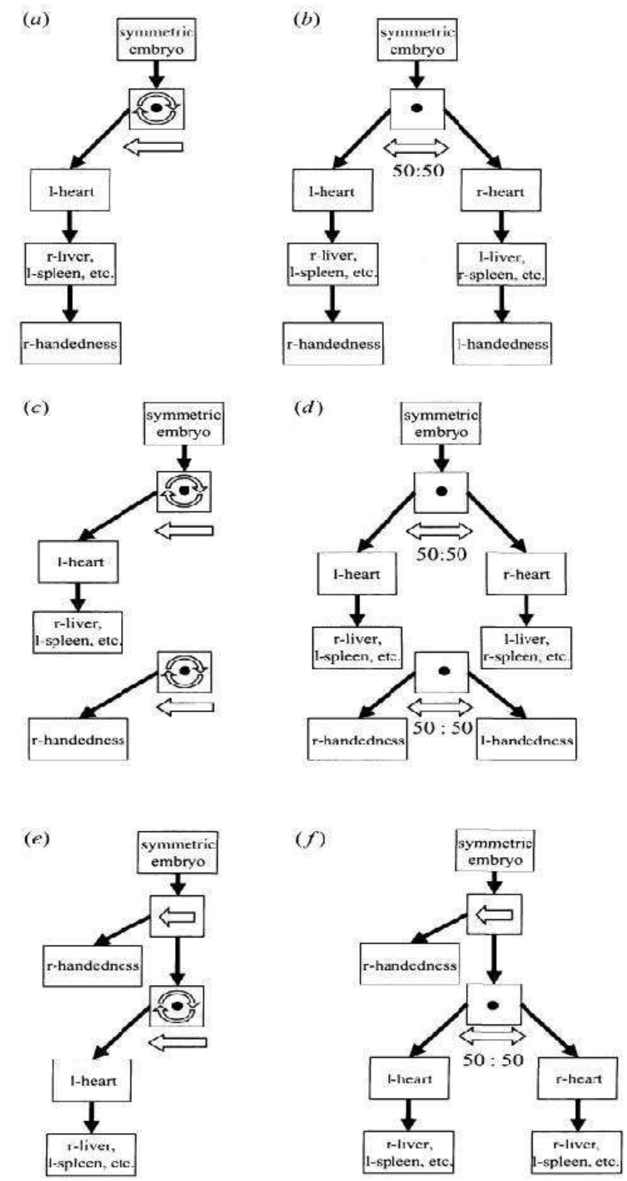

Figure A.1. Models on visceral organ and handedness orientations, from [13].

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