Negative effective mass density of acoustic metamaterial using dual-resonator spring-mass model

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Abstract

In this paper, we propose an acoustic metamaterial with a microstructure consisting of two internal resonators. The performance of this dual-resonator metamaterial is compared to the original single-resonator metamaterial. Analytical findings show that the dual-resonator metamaterial exhibits its negative effective mass density over a larger frequency spectrum, particularly at two distinctively asymptotic regions. The wave propagation phenomenon in the metamaterial is investigated using finite element simulation. Computational results reveal that the dual-resonator metamaterial is capable of attenuating wave propagation in a larger operating frequency. Practical applications like vibration control and blast mitigation are demonstrated and discussed.

1. Introduction

The research field in metamaterials continues to arouse tremendous attention among researchers worldwide. Recently, research interest has been extended to the acoustic counterpart besides the originally initiated electromagnetic metamaterials. Researchers investigating acoustic metamaterials are particularly interested in the unique properties of negative effective mass density and negative effective modulus, which are enacted by specially designed man-made microstructures [1-3].

The negative effective mass density can be exhibited in an acoustic metamaterial using lattice system consisting of mass-in-mass units [3-5]. It is found out that the effective mass density becomes frequency dependent and may become negative for frequencies near the resonance frequency of the internal mass. It has been shown that such system is capable of attenuating wave propagation [4]. However, one limitation of such system is that the negative effective mass density operates within a very narrow band gap region, specifically near the resonance frequency of the internal resonator. To overcome this limitation, we propose the use of two internal resonators as microstructure in an acoustic metamaterial. This paper presents the analytical findings of the new model, showing a larger operating frequency for negative effective mass, and demonstrates practical applications like vibration control and blast mitigation using finite element simulation.

2. Analytical model

We employ the use of a one-dimensional spring-mass model to achieve the negative effective mass effect. Analogous to the mass-in-mass unit based on [3-5], we further develop the microstructure into a mass-in-(mass-in-mass) unit, having 2 internal resonators. By considering the equations of motions for the individual masses and equating the entire behaviour of the system as an effective mass behaviour (Fig. 1a), we obtain the effective mass equation as in (1). It is easy to note that the effective mass, m_{EFF}, is dependent on the frequency, ω of the system. By considering the total static mass, $m_{ST} = m_1 +$

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$m_2 + m_3$ and resonance frequency of inner-most internal mass, $\omega_2^2 = k_2/m_2$, we can generate a typical dimensionless plot of m_{EFF}/m_{ST} against ω/ω_2 (Fig. 1b). It is evident that the negative effective mass region spans over a significant portion of the frequency spectrum, having asymptotic values at two distinctive regions. A study on the dispersion curves and the band gap structure of a similar multi-resonator mass-in-mass system has been done by [6].

$$m_{EFF} = m_3 + \frac{(m_1+m_2)k_1k_2 - m_1m_2k_1\omega^2}{(k_2-m_2\omega^2)(k_1+k_2-m_1\omega^2)-k_2^2} \tag{1}$$

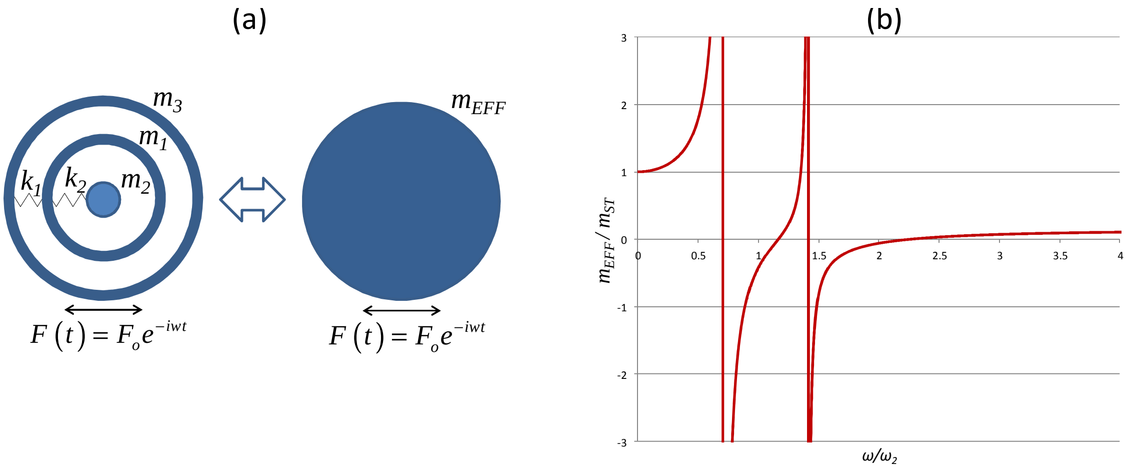

Fig. 1: (a) Dual-resonator spring-mass system and its effective mass model, (b) Plot of m_{EFF}/m_{ST} against ω/ω_2.

3. Finite element simulation

The performance of the proposed dual-resonator acoustic metamaterial is demonstrated using finite element simulation in practical applications like vibration control and blast mitigation. In vibration control case, we have an acoustic metamaterial consisting of 10 microstructure unit cells. At unit cell number one, an input excitation displacement of $D(0,t)$ = 0.0001 (sin $\omega_1 t$ + sin $\omega_2 t$) is given (ω_1=100rad/s and ω_2=300rad/s in this study). At unit cell number ten, the output response is obtained.

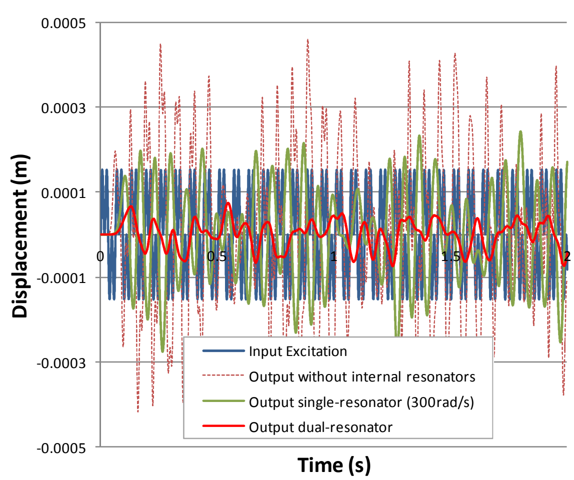

Fig. 2: Vibration control output responses using different microstructures.

Fig. 2 shows the output response comparison using various microstructures. It is clear that having a single-resonator metamaterial is unable to effectively attenuate the vibration wave. The dual-resonator metamaterial works well and is able to significantly attenuate the input excitation wave. In the blast mitigation case, the acoustic metamaterial is given a force controlled impact blast pulse as shown in Fig. 3a. Blast loading is characterised by a short time duration and relatively high load peak. Due to the blast pulse, stress wave with a wide frequency domain is generated. When internal resonators are absent in the material, a significant power amplitude over large frequency domain can be seen (Fig. 3b). However, when the proposed dual-resonator metamaterial is used to block the wave propagation, this results in substantial reduction in the power amplitude over the designed frequency range.

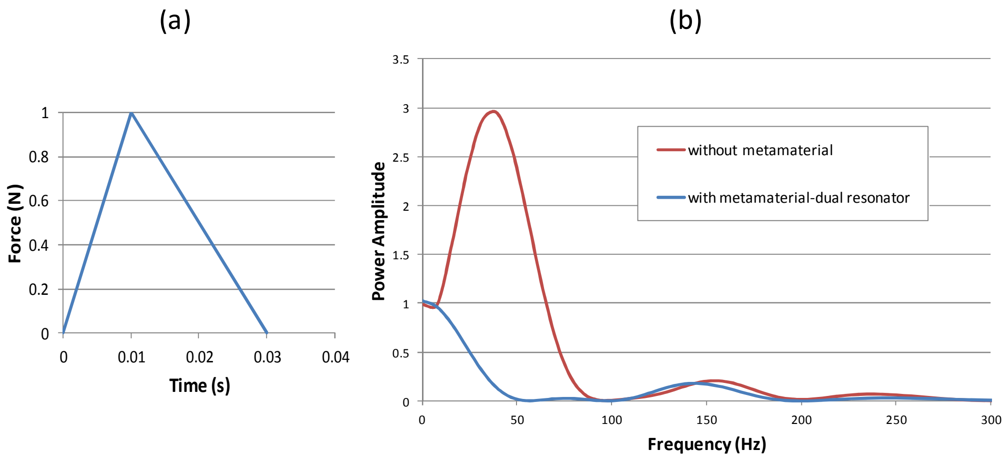

Fig. 3: (a) Input blast pulse, (b) Frequency domain by Fast Fourier Transform.

4. Conclusion

In this paper, we have proposed a dual-resonator acoustic metamaterial. This newly proposed model has an advantage of having its negative effective mass density exhibited over a larger frequency, as compared to the originally proposed single-resonator acoustic metamaterial. By examining the analytical model, it is clear that the effective mass becomes negative over two distinctive regions. Finite element simulation is used to demonstrate the practical application of the proposed model. In cases like vibration control and blast mitigation, the proposed dual-resonator acoustic metamaterial has proven to be effective in attenuating stress wave propagation over larger frequency range.

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