# Analysis of Acoustic Metamaterials - Acoustic Scattering Matrix and Extraction of Effective Parameters

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**Abstract**

To allow straightforward characterization of acoustic metamaterials, we present a method for extraction of complex S-parameters, based on the analogies that exist between propagation of electromagnetic and acoustic waves. We then propose the method for extraction of effective material properties of acoustic metamaterials, namely the bulk modulus and density. The proposed methods are validated through simulations in COMSOL Multiphysics and can be used as useful tool in design of various acoustic metamaterials.

## 1. Introduction

Electromagnetic (EM) metamaterials are artificial structures that exhibit electromagnetic properties generally not found in nature. A similar phenomenon can be observed in acoustics, based on the analogies between propagation of EM and acoustic waves. In this way, a concept of acoustic metamaterials can be developed, where effective bulk modulus and density can be tailored to exhibit arbitrarily small, large or even negative values. In this work we opted to characterize acoustic metamaterials using the scattering matrix [1] approach, as it is the basis of most methods for the analysis of EM metamaterials.

## 2. Extraction of complex S-parameters

Let us consider a slab of unknown acoustic medium with acoustic impedance $Z_x$ and transmission coefficient $\gamma$ inserted in a long acoustic duct filled with air, the acoustic impedance of which is $Z_0$, as shown in Fig. 1.

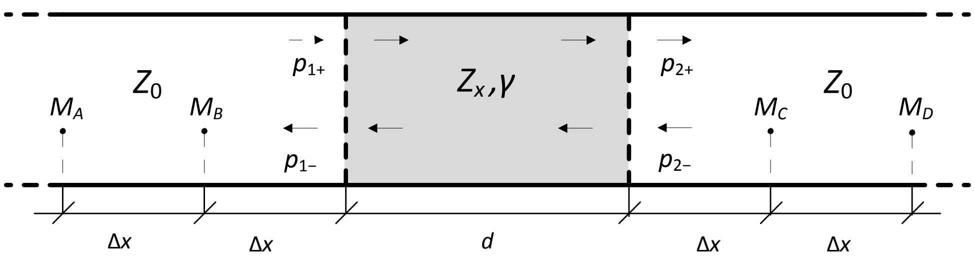


Fig 1. Extraction of complex S-parameters

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Under the assumption that the structure allows the propagation of plane waves only, the definition of acoustic scattering parameters, by analogy with their definition in electromagnetics, is the following:

$$\begin{bmatrix} p_{1-} \\ p_{2+} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} p_{1+} \\ p_{2-} \end{bmatrix} \tag{1}$$

where $p_{1+}$ and $p_{1-}$ denote the phasors of transmitted and reflected acoustic waves in air at the left boundary between the air and the unknown medium, and $p_{2+}$ and $p_{2-}$ denote the phasors of transmitted and reflected acoustic waves in the air at the right boundary between the unknown medium and air. As we are able to measure only the sum of the transmitted and the reflected wave at a certain point, the problem of calculating S-parameters amounts to the separation of measured values into their corresponding transmitted and reflected components. We measure acoustic pressure at 4 points, using 4 microphones located as shown in Fig. 1. For the sake of simplicity, the four distances are set to be identical and equal to $\Delta x$. The total pressures measured by the 4 microphones are given by:

$$p_A = p_{1+}e^{jk_0 2\Delta x} + p_{1-}e^{-jk_0 2\Delta x}, \tag{2}$$

$$p_B = p_{1+}e^{jk_0 \Delta x} + p_{1-}e^{-jk_0 \Delta x} \tag{3}$$

$$p_C = p_{2+}e^{-jk_0 \Delta x} + p_{2-}e^{jk_0 \Delta x}. \tag{4}$$

$$p_D = p_{2+}e^{-jk_0 2\Delta x} + p_{2-}e^{jk_0 2\Delta x} \tag{5}$$

where $k_0 = \omega/c$ is the wave number in air. Given this set of equations, we can compute the transmitted and reflected components of pressures, both in simulations as well as in measurements. In a general case this would be insufficient to calculate the whole scattering matrix, and it would be necessary to perform two sets of simulations/measurements – one with the source on the left side and one with the source on the right side of the structure. However, under the assumption that the slab of unknown medium constitutes a symmetrical and reciprocal system, one set of measurements is sufficient, and the complex S-parameters can be easily found from the matrix equation (1).

## 3. Extraction of complex effective material parameters

Effective material parameters of the acoustic metamaterial can be extracted from the scattering matrix using the similar method as in the case of EM metamaterials, [2-3]. Acoustic wave impedance $z$ and refractive index $n$ are calculated directly from the S-matrix by adjusting formulas according to electro-acoustical analogies, taking into account that Kramers-Kronig relation must be satisfied to ensure the continuity of extracted complex material parameters and the uniqueness of the solution. Effective material parameters, namely the bulk modulus ($B_{eff}$) and density ($\rho_{eff}$), are analogous to the reciprocal value of permittivity ($\varepsilon_{eff}^{-1}$) and permeability ($\mu_{eff}$), respectively, and can be calculated as

$$B_{eff} = \frac{z}{n}, \tag{6}$$

$$\rho_{eff} = nz. \tag{7}$$

The method was tested through simulations of the Helmholtz resonator field with air, performed in COMSOLS Multiphysics 4.0. The Helmholtz resonator (HR) is an acoustic metamaterial unit cell of the resonant type which exhibits a negative bulk modulus in a narrow frequency range [4]. It consists of a cavity of known volume with rigid walls connected to the host structure by a neck of known length. The HR exhibits a notch in a transmission characteristic in a frequency range where reciprocal bulk modulus is less than zero, as shown in Fig. 2.

Using the method described above effective acoustic material parameters of HR have been extracted and shown in Fig. 3 and in Fig. 4. It can be seen that the bulk modulus is less than zero in a narrow frequency range around 800 kHz, while out of this frequency range it is equal to the bulk modulus of

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air $B_{air}$=1.42·$10^5$ Pa. As expected, effective density also has a resonant-type response, but the amplitude of this resonance is not strong enough to result in negative effective density. Out of the resonant band, the effective density asymptotically approaches the value of density of air $\rho$=1.25 kg/$m^3$. The shape and values of the extracted material parameters correspond very well to the expected ones, confirming the validity of the approach presented in this paper.

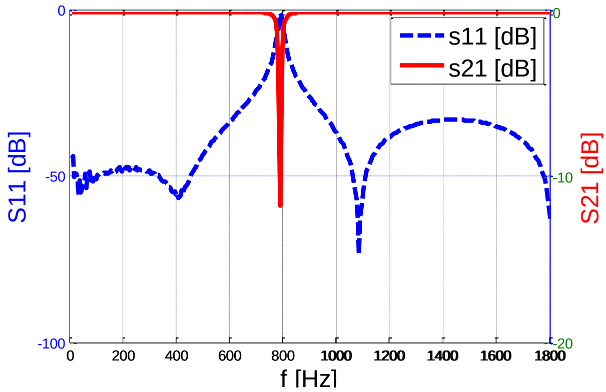


Fig 2. Acoustic S-parameters of Helmholtz resonator

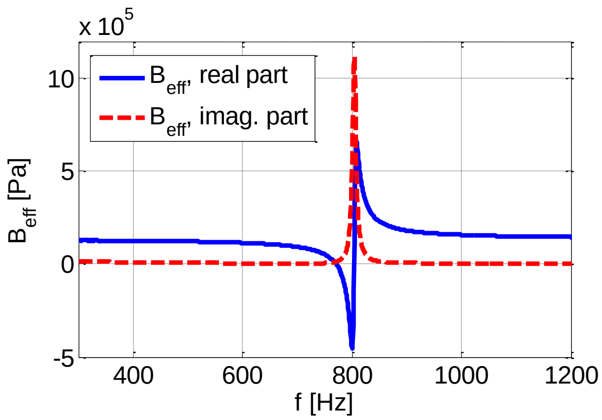


Fig 3. Effective bulk modulus of Helmholtz resonator

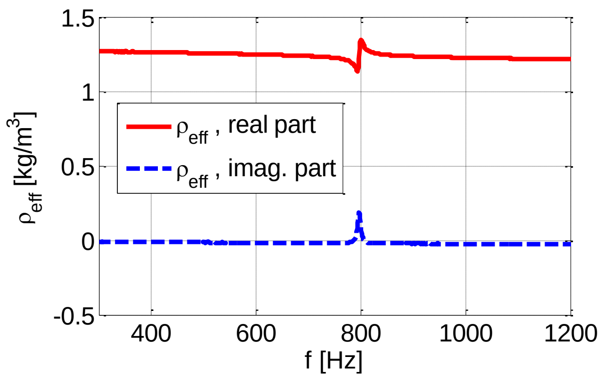


Fig 4. Effective density of Helmholtz resonator

## 4. Conclusion

Firstly, a method has been presented for the extraction of complex S-parameters from simulations or measurements of acoustic metamaterials. The obtained S-matrix is then used to extract effective material properties of acoustic metamaterials, namely the bulk modulus and density, thus providing a useful tool for the design of acoustic metamaterials.

### References

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