Exotic reflection of plane waves by anisotropic structures

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Abstract
An uniaxial anisotropic semi-infinite medium with arbitrary orientation of the anisotropy axis under tangential wave incidence is considered. A possibility of normal reflection of a plane wave is studied. The mathematical model describing such a possibility is presented and the results of numerical calculations are given and analyzed.

1. Introduction

In [1,2] has been described the penetration effect. In that work the possibility of transmitting a plane wave tangentially propagating under an interface between an isotropic and anisotropic lossless media through this interface has been given the mathematical ground. However in [1,2] it has not been considered behavior of a reflected wave. Note that the similar effect has been described in [3-5] also. But they have considered an oblique wave incidence on a metal film. The extraordinary transmission is generally described as being mediated by surface plasmons which are coherent oscillations of the metal’s conducting electrons at the surface of the metal.

Here we study the phenomena at an interface between two dielectric (isotropic and anisotropic) media and describe the behavior of a reflected wave by using the classic electromagnetic approach. Since the form of electromagnetic field within the anisotropic medium an electromagnetic field in a free space necessarily must has a longitudinal component of electric or magnetic field. Therefore in this problem at first it is necessary to analyze the next cases: a) a plane incident wave excites a surface wave; b) a reflected wave is reflected under an oblique angle; c) there is no penetration effect for the considered anisotropic medium. The study of all considered cases leads to the conclusions that a surface wave and a reflection under an oblique angle cannot be existence in the considered problem. However, absence of the penetration effect leads to violation of the continuity conditions at the interface. Since of this we support possibility of an exotic wave reflection (perpendicularly to the interface) and verify the boundary conditions. First of all this assumption allows us to satisfy the all boundary conditions. Moreover for today there are the experimental results describing a similar abnormal reflection by a metamaterial [6].
Here the results for TE-incident wave are presented (Fig.1). Analogous results are obtained for TM-wave also.

2. Statement of the problem

In this paper we consider a semi-infinite uniaxial anisotropic medium with an oblique orientation of the anisotropy axis described by permittivity dyadic

\[ \bar{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \tag{1} \]

and the scalar permeability \( \mu_0 \). Here it is studied the phenomena at interface between semi-infinite anisotropic medium and free space when a plane incident wave propagates tangentially to interface (Fig.1). Our main purpose is developing the mathematical model describing wave behavior in the considered case.

3. Infinite medium

**Extraordinary wave.** This wave contains \( E_{y'} \), \( E_{z'} \), \( H_{x'} \) field components. Note that the normal components of wavevectors are different for forward and backward waves in this case. These expressions are very unwieldy and are not written here. It is obtained also that Poyting vector contains \( y' \)- and \( z' \)-real components.

**Ordinary wave.** This wave contains \( E_{x''} \), \( H_{y''} \), \( H_{z''} \) components, the normal components of wavevectors are real and these are the same for forward and backward waves. It also found that Poyting vector contains \( y'' \)- and \( z'' \)-real components.

It means that any wave in free space that has nonzero tangential field components and nonzero tangential component of wavevector at interface between vacuum and the considered medium can excite a bulk wave within the medium. A surface wave or usual plane harmonic wave can be such a wave. We present the results of investigation for a plane wave.

4. Semi-infinite medium

As it is note above we support possibility of an exotic wave reflection perpendicularly to the interface and verify the boundary conditions. Let us consider this case in detail (Fig.1). The electromagnetic field in an isotropic medium is a superposition of two orthogonal waves. Moreover resulting field includes \( E_{y''} \), \( E_{z''} \), \( H_{x''} \) components, the wavevectors of the waves are perpendicular and the tangential to the interface, the tangential component \( k_y \) is equal to wavenumber in free space.

Certainly, it would be possible to assume also that mutually orthogonal waves can be obtained by using two different sources. However in such a case a normally incident wave should be reflected by the interface only, and a tangentially radiated wave should not interact with the interface.

**Extraordinary wave.** Taking into account above arguments we consider the case of tangential incidence and normal reflection of a wave. The boundary conditions for field components are written with taking into account is a surface change density. Physically a surface change in the considered dielectric anisotropic medium can exist due to polarization of molecules. The reflection and transmission coefficients correspondingly are

\[ R_2 = \frac{k_y (\varepsilon_{xx} \sin^2 \theta + \varepsilon_{zz} \cos^2 \theta) - k_y (\varepsilon_{xx} - \varepsilon_{zz}) \sin \theta \cos \theta}{k_y (\varepsilon_{xx} \sin^2 \theta + \varepsilon_{zz} \cos^2 \theta) - k_y (\varepsilon_{xx} - \varepsilon_{zz}) \sin \theta \cos \theta - \rho \omega \varepsilon_{xx} \varepsilon_{zz}} \]

\[ T_2 = \frac{k_y (\varepsilon_{xx} \sin^2 \theta + \varepsilon_{zz} \cos^2 \theta) - k_y (\varepsilon_{xx} - \varepsilon_{zz}) \sin \theta \cos \theta - \rho \omega \varepsilon_{xx} \varepsilon_{zz}}{k_y (\varepsilon_{xx} \sin^2 \theta + \varepsilon_{zz} \cos^2 \theta) - k_y (\varepsilon_{xx} - \varepsilon_{zz}) \sin \theta \cos \theta - \rho \omega \varepsilon_{xx} \varepsilon_{zz}} \tag{2} \]
Here \( \theta \) is an inclination angle of the anisotropy axis, \( k_{x} \), \( k_{z} \) are the wavevector components. It is seen from (2) that for semi-infinite medium the reflection and transmission coefficients are not dependent on frequency if the constitutive parameters have no frequency dispersion. The surface change density is:

\[
\sigma_{s} = \frac{k_{y} \left( \varepsilon_{zz} \sin \theta \cos \theta - \varepsilon_{zz} \sin^{2} \theta + \varepsilon_{zz} \cos^{2} \theta \right) - \varepsilon_{0} k_{y} \left( \varepsilon_{zz} \sin^{2} \theta + \varepsilon_{zz} \cos^{2} \theta \right)}{k_{y} \left( \varepsilon_{zz} \sin \theta \cos \theta - \varepsilon_{zz} \sin^{2} \theta + \varepsilon_{zz} \cos^{2} \theta \right) + \omega \varepsilon_{zz} \varepsilon_{zz} \sin \theta \cos \theta}
\]  

(3)

Thus all boundary conditions are satisfied and moreover, in spite of exotic kind, the presented hypotheses are not contradicted to exiting physical theories.

**Ordinary wave.** Analogous results are obtained for an ordinary wave if it is assumed existence displacement currents.

5. Numerical calculations

The results of the numerical calculations for the semi-infinite anisotropic medium with the constitutive parameters \( \varepsilon_{xx} = 2.12 \varepsilon_{0}, \varepsilon_{zz} = 0.99 \varepsilon_{0} \) are presented in Fig.2. In Fig.2a the intensity reflection coefficient is shown by the solid line and the intensity transmission coefficient is shown by the dash line. It is seen that near \( \theta = 0 \) the method is not calculate correctly as denominator in (2) is very large.

![Fig.2](image)

In Fig.2b the surface change density is presented for the considered case. It is seen that decrease of the transmission coefficient leads to not only increase of the reflection coefficient and also to increase of the surface change.

It also is investigated the dependences of the refraction angle and the angle of Poyting vector propagation within the anisotropic medium. The conditions for which angle of Poyting vector propagation is negative are found.

5. References


