Duality in 2D optical nanocircuits

J. D. Baena[1], J. D. Ortiz[1], R. Marqués[2], V. Delgado[2]

[1]Department of Physics, Universidad Nacional de Colombia, Bogota, Colombia
Emails: jdbaenad@unal.edu.co; jdortizc@unal.edu.co
[2]Department of Electronics and Electromagnetism, Universidad de Sevilla, Seville, Spain
Emails: marques@us.es; vdelgado@us.es

Abstract

A duality relation between the impedances of a single original element and its complementary element is deduced. It is also demonstrated that this duality property holds also for parallel or series associations of several circuit elements. This work can help to design 2D optical circuits with dual properties.

1. Introduction

A few years ago, Nader Engheta and co-workers proposed the theoretical possibility of extending the circuit theory to the optical spectrum by arranging plasmonic and non-plasmonic elements into a nanostructure of size much smaller than the wavelength [1]. This idea is nicely summarized by saying that pieces with positive permittivity behaves like nanocapacitors, while those with negative permittivity like nanoinductors. Even nanoresistors enter to play a role because the permittivity must have a certain imaginary part. In order to encapsulate all elements and make their impedance independent on the shape of the external field, they also proposed to cover all pieces with thin slabs of Epsilon Near Zero (ENZ) and Very Large Epsilon (VLE) media [2]. ENZ forces the displacement current to be parallel to the boundary, so that it can be considered as an insulator, while VLE forces the displacement current to be orthogonal to the boundary, so that it is like an electrode. Separately, at least since 40's, the duality property has played an important role in conventional planar electronic circuits [3]. For instance, it has been used for designing complementary filters starting from original filters, in such a way that the passbands/stopbands change to stopbands/passband over the same frequency range. That duality property was based on the duality of the electric and magnetic fields and the interchange between electric and magnetic walls. However, in the framework of optical nanocircuits the electric walls cannot exist because of the plasmonic behaviour of metals at so high frequencies. In this paper we try to bring the duality principle to the frame of 2D optical nanocircuits, with the hope that it could be used in future designs of dual or complementary structures.

2. The Theorem of Complementarity

Let us assume that the quasi-electrostatic approximation is valid and we are dealing with a problem without free charge, so that $\nabla \cdot \mathbf{D} = 0$ and $\nabla \times \mathbf{E} \approx 0$, where **D** and **E** are going to be considered related by $\mathbf{D} = \varepsilon \mathbf{E}$ and the permittivity ε is isotropic and homogeneous by pieces. Let us also assume translation symmetry along the z-axis, so that the problem is 2D. We have demonstrated in [4] that, given a known solution **E** of a certain problem, we can define a complementary problem whose field **E**' and permittivities are obtained from the original one by using

$$\mathbf{E}'_i = k_i \hat{\mathbf{z}} \times \mathbf{E}_i \;; \quad k_i / k_j = \varepsilon_i / \varepsilon_j = \varepsilon'_j / \varepsilon'_i \tag{1}$$

where $i, j = 1...n$ are indexes indicating two particular regions among n regions. Physically, it means that the field is rotated by 90° and rescaled by the factor k_i, and the regions are filled with new values of permittivities ε'_i. It can be demonstrated that this theorem still holds when some free current density $\mathbf{J} \neq 0$ is present, but only if we choose the complementary source $\mathbf{J}'_i = (\varepsilon'_i / \varepsilon_i) k_i \hat{\mathbf{z}} \times \mathbf{J}_i$.

3. Duality for a single circuit element

In Fig. 1 examples of a single circuit element (a) and its complementary circuit (b) are shown. In passing from the original to the complementary circuit, the shape of the boundaries is kept the same and the permittivities are changed by using relations of Eq. (3). Different colors mean different values of the permittivity filling each region: white for finite and non-zero permittivity, red for ENZ medium, and green for VLE medium. For the original structure, region #1 (white) corresponds to the core of the element, while regions #2 and #3 are connector and insulator, respectively. Dashed lines represent the displacement current density flowing along the circuit. By using (1), we get

$$\mathbf{E}'_1 = k_1 \hat{\mathbf{z}} \times \mathbf{E}_1 \;; \quad \mathbf{E}'_2 = k_2 \hat{\mathbf{z}} \times \mathbf{E}_2 \;; \quad \mathbf{E}'_3 = k_3 \hat{\mathbf{z}} \times \mathbf{E}_3 \tag{2}$$

$$k_1 / k_2 = \varepsilon_1 / \varepsilon_2 = \varepsilon'_2 / \varepsilon'_1 \;; \quad k_2 / k_3 = \varepsilon_2 / \varepsilon_3 = \varepsilon'_3 / \varepsilon'_2 \;; \quad k_3 / k_1 = \varepsilon_3 / \varepsilon_1 = \varepsilon'_1 / \varepsilon'_3 \tag{3}$$

By taking ε'_1 as the free unknown of (3), we get $\varepsilon'_2 = (\varepsilon_1 / \varepsilon_2)\varepsilon'_1$ and $\varepsilon'_3 = (\varepsilon_1 / \varepsilon_3)\varepsilon'_1$. And taking k_1 as the free unknown of (3), we can express k_2 and k_3 in terms of k_1 and after replace them into the formulas (2) in order to get the electric fields $\mathbf{E}'_1 = k_1 \hat{\mathbf{z}} \times \mathbf{E}_1$ (finite), $\mathbf{E}'_2 = (\varepsilon_2 / \varepsilon_1) k_1 \hat{\mathbf{z}} \times \mathbf{E}_2 \to \infty \cdot 0$ (finite), and $\mathbf{E}'_3 = (\varepsilon_3 / \varepsilon_1) k_1 \hat{\mathbf{z}} \times \mathbf{E}_3 \to 0 \cdot [\text{finite}] \to 0$. Physically, in the region #1 the field has been rotated 90° and rescaled by some factor, and regions #2 and #3 have interchanged their roles (in the complementary circuit, #2 is insulator and #3 is connector). It is obvious that the impedance for ENZ regions tends to infinity and the impedance for VLE regions tends to zero, thus we avoid their corresponding discussions. Regarding the region #1, the core of the circuit, we can define the impedances for the original and the complementary structures and multiply them as follows:

$$ZZ' = \frac{V}{I}\frac{V'}{I'} = \frac{-\int_{B_1} \mathbf{E}_1 \cdot \hat{\mathbf{l}} dl}{j\omega\varepsilon_1 \iint_{A_1} \mathbf{E}_1 \cdot \hat{\mathbf{n}} da} \; \frac{-\int_{A_1} \mathbf{E}'_1 \cdot \hat{\mathbf{l}}' dl'}{j\omega\varepsilon'_1 \iint_{B_1} \mathbf{E}'_1 \cdot \hat{\mathbf{n}}' da} = \frac{-1}{\omega^2 \varepsilon_1 \varepsilon'_1 h^2} \tag{4}$$

where in the last step we have used the facts that $\mathbf{E}'_1 \cdot \hat{\mathbf{l}}' = k_1 \mathbf{E}_1 \cdot \hat{\mathbf{n}}$ and $\mathbf{E}'_1 \cdot \hat{\mathbf{n}}' = k_1 \mathbf{E}_1 \cdot \hat{\mathbf{l}}$., and h is the thickness of the structure in the z direction (ideally it is infinite, but not for real samples). Note the similarities with the duality property in electronics $ZZ' = \eta_0^2 / 4$ [3], being η_0 the vacuum impedance.

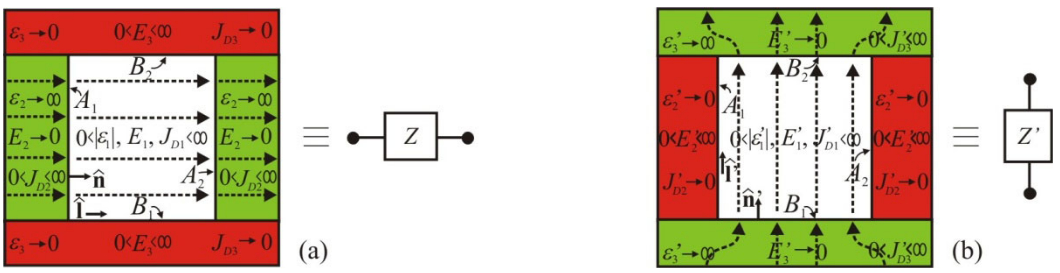

Fig. 1: Example of an original circuit element (a) and its corresponding complementary circuit (b).

ISBN 978-952-67611-2-1

4. Duality for series and parallel connections

Let us consider the structures shown in Fig. 2. By applying Eq. (3) to the interfaces #1-connector and the #2-connector and dividing them, we get $k_1/k_2 = \varepsilon_1/\varepsilon_2 = \varepsilon'_2/\varepsilon'_1$. By going from the original circuit (Fig. 2(a)) to the complementary circuit (Fig. 2(b)), the connectors (green) and insulators (red) interchange their roles and so also positions which is in accordance with the rotation of 90º applied to the field. By using (3) and the fact that the flux of **D** through the boundaries A_2 and A_3 of the original problem (Fig. 2(a)) must be equal, we can easily write the next calculation

$$\int_{A_3} E'_2 dl = k_2 \int_{A_3} E_2 dl = (k_1\varepsilon_2/\varepsilon_1)\int_{A_3} E_2 dl = (k_1/\varepsilon_1)\int_{A_3} \varepsilon_2 E_2 dl = (k_1/\varepsilon_1)\int_{A_2} \varepsilon_1 E_1 dl = \int_{A_2} E'_1 dl \quad (5)$$

Looking at the ends of this equation, it is clear that the voltages in regions #1 and #2 of Fig. 2(b) are equals, so that the complementary structure works like a parallel connection. Reciprocally, if we were started from a parallel connection then it would turn into a series connection. Finally, let us imagine that we take the complementary structure with $\varepsilon'_1 = \varepsilon_2$ and $\varepsilon'_2 = \varepsilon_1$ (a simple interchange of the two media in cores). Applying (4) to each region we get $Z_1 Z'_1 = Z_2 Z'_2 = -(\omega^2 \varepsilon_1 \varepsilon_2 h^2)^{-1}$. And the product of the effective impedances of the two structures is $Z_{eff} Z'_{eff} = (Z_1 + Z_2)/(Z'^{-1}_1 + Z'^{-1}_2) = -(\omega^2 \varepsilon_1 \varepsilon_2 h^2)^{-1}$ which interestingly coincides with (4). It is now obvious that the duality property is inherited by more complex associations made of several circuit elements.

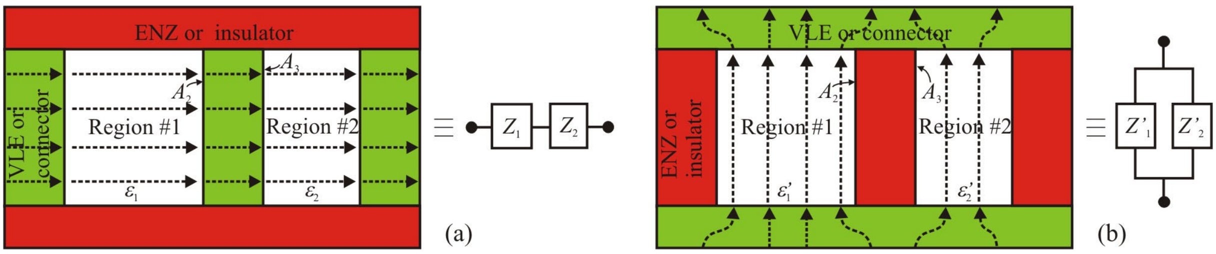

Fig. 2: Example of an original structure made of two elements (a) and its complementary structure (b).

5. Conclusion

The duality principle previously used in low frequency electronics, RF and microwaves [3], have been extended to the new branch of physics called optical nanocircuits [2]. Equation (4) summarizes the duality property for a single 2D nanocircuit. Besides, we demonstrated that a series/parallel connection of the original structure is replaced by a parallel/series connection in the complementary structure. It is worth to note that, although Fig. 1 and Fig. 2 show rectangular examples, the demonstration of equations (4) and (5) are also valid for structures with general curved shape of the boundaries. We think that these ideas open the door to the design of 2D optical nanocircuits with dual responses.

References

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