Finite element analyses for random laser action in metallic disordered structures

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Abstract

This paper presents the analyses of laser action in two-dimensional disordered structures consisting of metallic cylinders by means of a finite element method. Metallic cylinders reflect light waves completely on their surfaces, and then, random multiple scattering in the disordered structures is expected to become extremely intense. Hence, random lasers in the random systems of metallic cylinders are expected to be extremely low-threshold lasers. Metallic cylinders and active medium in the random systems are modeled by Drude model and negative value in imaginary part of relative permittivity, respectively. The lasing threshold in the random structures of metallic cylinders becomes much lower than those in random structures of dielectric cylinders.

1. Introduction

Random lasers [1–3] are lasing phenomena in disordered structures usually composed of dielectric materials. Such lasing phenomena occur from random multiple scattering and interference effect of scattered light in disordered structures. Random lasers realize low-threshold laser action [1, 3], broad-angle emission and broadband lasing. The above lasing properties are characteristic, and hence, random lasers are expected to be applied to advanced laser devices as displays, thermal sensors, etc.

Noble metals, as silver and gold, have highly reflective surfaces because of the existence of free electrons. In random structures composed of the noble metals, it is expected that the multiple scattering becomes more intense than the scattering in dielectric random structures. Additionally, light waves are strongly confined in the spaces among the noble metals in the random structures. By filling the spaces with active medium, light waves can be confined in the active medium. Hence, the noble metals are ideal materials for low-threshold random lasers. Random lasers in systems composed of noble metals are experimentally investigated actively. Lawandy reported the increase of surface enhanced scattering [4]. Kang observed random lasing enhanced by resonant surface plasmon in gold nanoshell/water solution [5] and showed the relation between pump power and signal intensity. Kumar et al. investigated random laser in dielectric-metal-dielectric surface plasmon waveguide [6] and showed the relationship between lasing threshold and metal thickness in the dielectric-metal-dielectric waveguide. However, no discussions are found on the threshold of random lasers in random systems consisting of metallic cylinders.

In this paper, we simulate random laser based on nanometer-sized silver cylinders by means of finite element method (FEM). The relative permittivity of the noble metals are written as Drude model [7] and the population inversion density of active medium is modeled by negative value in imaginary part of relative permittivity. The thresholds of metallic random lasers are compared with those of dielectric ones. The thresholds of metallic random lasers become much lower than those of dielectric ones.

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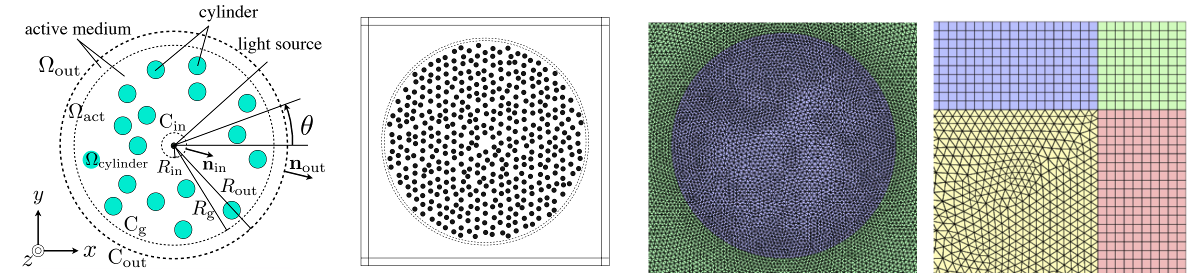
2. Analysis models and finite element method

Figs. 1(a) and 1(b) show an image of the random system and one of the samples analyzed in this study, respectively. Solid circles in Fig. 1(b) represent cylinders in random systems, and the filling factor of the cylinders are 30%. Disordered index which indicates the amount of disorder is $|\Delta \mathbf{x_r}|_{\max}/a = 1$ that is explained in [3].

FEM are used to simulate laser action precisely. Harmonic oscillations of electromagnetic waves and TM mode are assumed. Perfectly matched layers [8] are used to simulate light scattering in an open region. Finite elements are shown in Figs. 1(c) and 1(d).



(a) An illustration of random system. (b) An analysis model. (c) Finite element meshing in a cylinder. (d) Finite element meshing in PML.

Fig. 1: Illustration, analysis model, and finite element meshing.

Active medium is modeled by negative value in imaginary part of relative permittivity $-\gamma$. Metallic cylinders are expressed by the Drude model. Relative permittivities in each region are given as follows:

$$\epsilon(\mathbf{x}) = \begin{cases} 1.0 + \mathrm{i}(-\gamma) & \mathbf{x} \in \Omega_{\rm act} \\ 1 - \frac{\omega_{\rm p}^2}{\omega^2} & \mathbf{x} \in \Omega_{\rm cylinder} \\ 1.0 & \mathbf{x} \in \Omega_{\rm out} \end{cases}, \tag{1}$$

where $\Omega_{\rm act}$ is the space among the cylinders, $\Omega_{\rm cylinder}$ is the region in the cylinders, and $\Omega_{\rm out}$ is open region which is outside of random systems. We assume silver as a material of cylinders and a plasma frequency used in this analysis is $\omega_{\rm p} = 1.37 \times 10^{16}$ [s^{-1}]. The radii of cylinders are assumed to be 100 [nm]. Then, normalized plasma frequency becomes 0.72731.

We define the amplification factor A as the ratio of the fluxes of the Poynting vectors of light, directed outward of the dielectric system, between the excited state ($\gamma > 0$) and non-excited state ($\gamma = 0$), as follows:

$$A = \frac{\int_{\rm C_{out}} \langle \mathbf{S} \rangle \cdot \mathbf{n}_{\rm out}\, dl\, |_{\gamma \geq 0}}{\int_{\rm C_{out}} \langle \mathbf{S} \rangle \cdot \mathbf{n}_{\rm out}\, dl\, |_{\gamma = 0}}, \tag{2}$$

where $\rm C_{out}$ is the large outermost circle in Fig. 1(a), $\mathbf{n}_{\rm out}$ is unit outward normal vectors to the circle $\rm C_{out}$, and $\langle \mathbf{S} \rangle$ is Poynting vectors in the following time-averaged form:

$$\langle \mathbf{S} \rangle = \mathrm{Re}\left(\frac{\mathbf{E} \times \mathbf{H}^*}{2} \right), \tag{3}$$

where $\mathbf{E}$ and $\mathbf{H}^*$ denote the electric field and the complex conjugate of the magnetic field. The light flux is calculated by a line integral of the Poynting vector along the circle $\rm C_{out}$.

3. Results

We compute amplification factor of light waves emitted from a metallic disordered structure by changing frequency and population inversion density for the ranges $0.250 \leq \omega a/2\pi c \leq 0.275$ and $0.00 \leq \gamma \leq 0.005$ and show it in Fig. 2(a). Fig. 2(b) shows computed amplification factors in the case of dielectric cylinders with relative permittivity 4. We can clearly observe that lasing phenomena in a metallic

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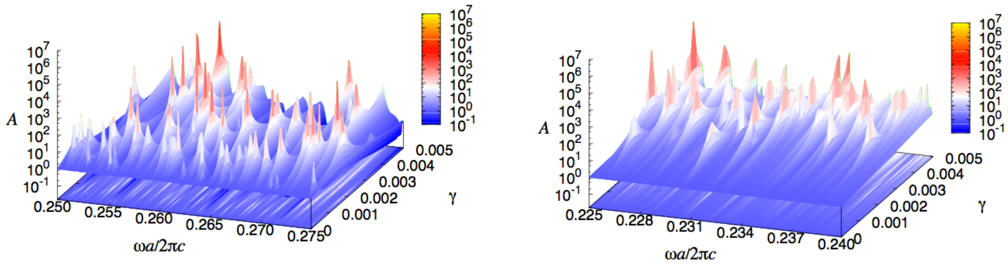
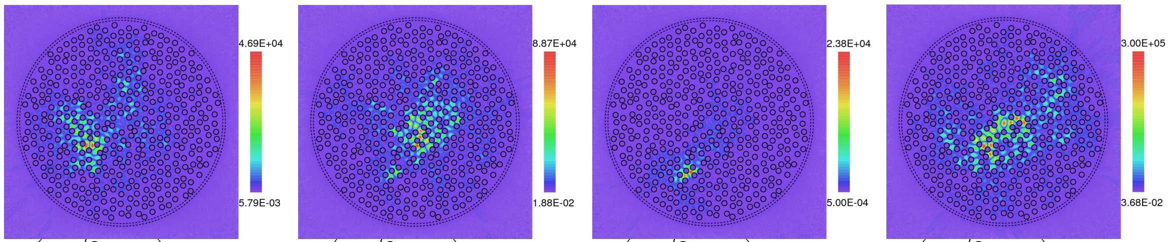
disordered structure start to occur at smaller values of γ than those in a dielectric one. The threshold of random lasing in a metallic disordered structure becomes extremely low. Fig. 3 shows electric amplitude distributions of lasing states occurring at lowest four γ. Light waves strongly confined in the spaces among the metallic cylinders and do not penetrate into the cylinders.



(a) Metallic disordered structure. (b) Dielectric disordered structure .

Fig. 2: Analysis model and finite element meshing.



(a) $(\omega a/2\pi c, \gamma) = (0.2659292, 0.000150)$. (b) $(\omega a/2\pi c, \gamma) = (0.2713668, 0.000160)$. (c) $(\omega a/2\pi c, \gamma) = (0.2707160, 0.000175)$. (d) $(\omega a/2\pi c, \gamma) = (0.2548204, 0.000205)$.

Fig. 3: Electric amplitude distributions of lasing states in a metallic disordered structure (sample 1).

4. Conclusions

Random lasing phenomena in disordered structures consisting of metallic cylinders are precisely simulated by using finite element method. The threshold of random lasers in disordered structures of metallic cylinders are newly revealed by the comparison with the threshold of random lasers in disordered structures of dielectric cylinders. The lasing threshold of metallic random lasers becomes much smaller than that of dielectric random lasers.

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