Nonlinear Beams and Field Concentrators with Hyperbolic Metamaterials

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Abstract

The hybrid analytical-numerical method [1], is developed for modelling beam propagation in non-linear field concentrator, based on hyperbolic metamaterial, consisting of two cylinders. External cylinder is linear, while internal one is nonlinear. The new version of complex geometrical optics (CGO) is used for describing the ray propagation in linear region, while in nonlinear region, full-wave nonlinear electromagnetic solution (FWNES) is applied, and the proper method of matching both solutions is developed. When the intensities of the incident electromagnetic beams reach some threshold, the switching to the highly nonlinear regime occurs.

1. Introduction

In modeling (linear) "electromagnetic black hole" (linear concentrator), intuitively clear method of (real) geometrical optics (GO) was applied [2]. In this paper, we develop a new method suitable for simulation of advanced nonlinear field/energy concentrators [1], based now on hyperbolic metamaterials. The concentrator includes a layered cylindrically symmetrical hyperbolic media consisting of alternating very thin layers of metal (semimetal or doped semiconductor) with a negative permittivity and dielecttric ones with a positive permittivity. The developed method (useful also for space plasma) includes the new version of CGO [1], used in external cylinder, FWNES for the inner nonlinear cylinder and their matching. New physical effect of "superfocusing" and nonlinear focusing switching is searched for hyperbolic metamaterials, and possible applications are discussed.

2. Formulation of the problem

When the thicknesses of the alternating (isotropic) layers are $d_{1,2}$, (containing metal/semi-metal or semiconductor and dieletric, respectively), the effective tangential and normal permittivities $\varepsilon_{t,n}$ are

$$\begin{gathered}\varepsilon_t = (\varepsilon_1 d_1 + \varepsilon_2 d_2)/d > 0;\ \ \mathrm{Re}(\varepsilon_t) > 0;\ \ 1/\varepsilon_n = (1/d)(d_1/\varepsilon_1 + d_2/\varepsilon_2); \\ \mathrm{Re}(\varepsilon_n) < 0;\ d \equiv d_1 + d_2;\ \ \mathrm{Re}(\varepsilon_1) < 0,\ \ \mathrm{Re}(\varepsilon_2) > 0\end{gathered} \tag{1}$$

The similar formulas are used for the averaging in cylindrical region $R_c \le r \le R_0$, while the internal nonlinear region $r \le R_c$, layers with negative permittivity include saturating Kerr nonlinearity

$$\varepsilon_{NL} = \varepsilon_1 + \alpha_1 |E|^2 /(1+\alpha_2 |E|^2), \ \ \mathrm{Re}(\varepsilon_1) < 0 \qquad (2a)$$

and $\alpha_{1,2}$ are corresponding constants. We suppose that given rays presenting electromagnetic waves propagate in the region $r > R_0$ ("surrounding media") and fall on the (linear) cylinder $R_c \le r \le R_0$, where the CGO is used. For the mode (E_r, E_φ, H_z) in the internal region $r \le R_c$, "full-wave" electromagnetic equation is

$$(1/r)(\partial/\partial r)(r\beta_\theta \partial H_z/\partial r) + (\beta_r/r^2)(\partial^2 H_z/\partial\theta^2) + k_0{}^2 H_z = 0,\ \beta_{r,\theta} = 1/\varepsilon_{r,\theta},\ k_0 = \omega/c \qquad (2b)$$

while the values $\varepsilon_{r,\theta}$ are introduced instead of $\varepsilon_{n,t}$, determined in eq. (1), respectively.

3. Schematic description of the analytical and numerical methods

Let us put in the region $R_c \le r \le R_0$, $H_z = A\exp[iS(r,\theta)]$, where $S(r,\theta)$ is eikonal, while effective momentum components are

$$p_r = \partial S/\partial r,\ p_\theta = \partial S/\partial\theta,\ dS = p_r dr + p_\theta d\theta \qquad (3)$$

Putting (3) into (2b), we get dispersion equation in the approximation of CGO

$$D = p_r^2 \beta_\theta - (p_r/r)\beta_\theta + (\beta_r/r^2) p_\theta^2 - k_0^2 = 0. \qquad (4)$$

The equations describing ray propagation in the CGO approximation [1] (now temporal dispersion of the plasma-like media (semi-metal or semiconductor) is accounted for), has the form (5)

$$dr/dt = -\mathrm{Re}(D_{p_r}/D_\omega),\ d\theta/dt = -\mathrm{Re}(D_{p\theta}/D_\omega),\ dp_r/dt = (D_r/D_{p_r})\,\mathrm{Re}(D_{p_r}/D_\omega),$$
$$p_\theta = -k_0\rho \qquad (5)$$

$D_r, D_{p_r}, D_{p_\theta}, D_\omega$ are partial derivatives of D from eq. (4), ρ is "impact distance" for corresponding ray incident from the region $r > R_0$ on the cylinder $r = R_0$. Inside the nonlinear region $r \le R_c$, we put

$$H_z = \sum_{m=-N/2+1}^{N/2} A_m(r)\exp(-im\theta) \qquad (6)$$

(N is number of the Fourier modes accounted for). No averaging is used in nonlinear region, and nonlinear wave equation for the amplitude of mth harmonic (6) takes the form (7)

$$(1/r)(d/dr)(r\beta(r)dA_m/dr) - (m^2\beta(r)/r^2)A_m + \Phi_m(r) = 0,\ \beta(r) = 1/\varepsilon, \qquad (7)$$

while dependence on r in $\beta(r)$ accounts for jumps of ε between alternation layers and $\Phi_m(r)$ is angular Fourier harmonic of the nonlinear term determined by eq. (8), namely

$$\Phi(r,\theta) = (1/r)(\partial/\partial r)(r\delta\beta\, \partial H_z/\partial r) + (1/r^2)(\partial/\partial\theta)(\delta\beta\, \partial H_z/\partial\theta),\ \delta\beta = 1/\varepsilon_{NL} - \beta(r)\,. \qquad (8)$$

Proper boundary conditions at $r=0$ are added. Idea of matching CGO and FWNES at $r = R_c$, is based on the "local quasiplanar" approximation for the cylindrical Fourier modes at $r = R_c - 0$. Then, the obtained system of equations for nonlinear Fourier modes A_m is solved by means of especially developed iteration-sweep method (ISM), including integro-interpolation method.

4. Results of the modelling

The interference of electromagnetic waves in inner nonlinear cylinder, computed, using FWNES, is shown in Figs. 1 a, b input amplitude below and above threshold value, respectively.

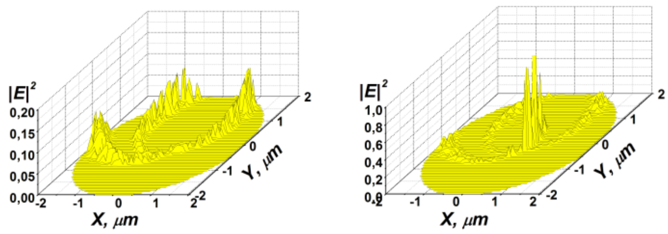

Fig. 1 Interference of electromagnetic waves in nonlinear inner cylinder $r \le R_c$ for amplitude of (each of three) incident beams below (left) and above (right) threshold value.

The first important result is a necessity of FWNES in each of the thin alternating layers in the internal cylinder, while "method of homogenization" is inapplicable. The second result is a switching to highly nonlinear regime with strong focusing inside a nonlinear region, when amplitude of input beam(s) reaches some threshold value (Fig. 1, right).

5. Conclusions

The new method, including CGO in linear region, FWNES in nonlinear region and their matching with (especially derived) proper boundary conditions is developed for a nonlinear cylindrical hyperbolic field concentrator. It is shown that (1) it is necessary to use FWNES inside nonlinear hyperbolic metamaterial and "method of averaging" is not enough accurate and (2) a switching to highly nonlinear regime with strong focusing is possible, when amplitude of input beam(s) reaches some threshold value. Strongly nonlinear focusing may happen not only on a boundary between linear and nonlinear regions, but inside a nonlinear cylinder. The position of a nonlinear focusing etc. are under detail investigation now. Application can include harmonic generation, sensors etc., from THz to IR range.

References

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