Polychronakos fractional statistics with a complex-valued parameter

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A generalization of quantum statistics is proposed in a fashion similar to the suggestion of Polychronakos [Phys. Lett. B 365, 202 (1996)], with the distribution function given by \( f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/T} - \alpha} \), where \( T \) is temperature, \( \mu \) is the chemical potential, and \( \varepsilon \) is the energy of the respective level. The parameter \( \alpha \) varies between \(-1\) (fermionic case) and \(+1\) (bosonic case). However, unlike the original formulation, it is suggested that intermediate values are located on the unit circle, \( \alpha = e^{i\pi \nu} \), but not on the real axis. In doing so, in particular, one can avoid the case \( \alpha = 0 \) corresponding to the Boltzmann statistics, which is not a quantum one. Such a defined statistics has a seeming drawback as it requires that some physical quantities, like energy or chemical potential, are complex. This is however not a problem as, for instance, complex-valued energy is usually connected to some dissipative processes [G. E. Cragg and A. K. Kerman, Phys. Rev. Lett. 94, 190402 (2005)]. Moreover, approaches involving complex chemical potential have a vast application domain, ranging from quantum chromodynamics [I. M. Barbour et al., Nucl. Phys. B (Proc. Suppl.) 34, 311 (1994)] to the physics of semiconductors [P. K. Chakraborty et al., J. Phys. Chem. Solids 64, 2191 (2003)]. In the work, a system of harmonic oscillators is analyzed. Several cases are considered in detail, namely, the limits of \( \nu \to 0 \) and \( \nu \to 1 \) reproducing small deviations from the Bose and Fermi statistics, respectively. Also, the case of a non-conserving number of excitations, which can be defined as \( \text{Re} \mu = 0 \), is studied. Thermodynamic quantities of these systems are calculated.