Polychronakos fractional statistics with a complex-valued parameter

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A generalization of quantum statistics is proposed in a fashion similar to the suggestion of Polychronakos [Phys. Lett. B **365**, 202 (1996)], with the distribution function given by $f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/T} - \alpha}$, where T is temperature, μ is the chemical potential, and ε is the energy of the respective level. The parameter α varies between -1 (fermionic case) and $+1$ (bosonic case). However, unlike the original formulation, it is suggested that intermediate values are located on the unit circle, $\alpha = e^{i\pi\nu}$, but not on the real axis. In doing so, in particular, one can avoid the case $\alpha = 0$ corresponding to the Boltzmann statistics, which is not a quantum one. Such a defined statistics has a seeming drawback as it requires that some physical quantities, like energy or chemical potential, are complex. This is however not a problem as, for instance, complex-valued energy is usually connected to some dissipative processes [G. E. Cragg and A. K. Kerman, Phys. Rev. Lett. **94**, 190402 (2005)]. Moreover, approaches involving complex chemical potential have a vast application domain, ranging from quantum chromodynamics [I. M. Barbour et al., Nucl. Phys. B (Proc. Suppl.) **34**, 311 (1994)] to the physics of semiconductors [P. K. Chakrabortya et al., J. Phys. Chem. Solids **64**, 2191 (2003)]. In the work, a system of harmonic oscillators is analyzed. Several cases are considered in detail, namely, the limits of $\nu \to 0$ and $\nu \to 1$ reproducing small deviations from the Bose and Fermi statistics, respectively. Also, the case of a non-conserving number of excitations, which can be defined as $\operatorname{Re}\mu = 0$, is studied. Thermodynamic quantities of these systems are calculated.