Complete Classification of 1D Gapped quantum Phases

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Abstract

Quantum many-body systems divide into a variety of phases with very different physical properties. The question of what kind of phases exist and how to identify them seems hard especially for strongly interacting systems. Here we provide a complete answer to this question for 1D gapped interacting quantum spin systems. Based on the local unitary equivalence relation between shortrange correlated states in the same phase, we classify all possible gapped quantum phases in 1D. We find that in the absence of any symmetry all states are equivalent to trivial product states, which means that there is no topological order in 1D. However, if certain symmetry is present, many 1D gapped phases exist with different symmetry breaking orders and/or symmetry protected topological orders. The symmetry breaking orders are completely classified by the subgroups of the symmetry group and the symmetry protected topological orders are completely classified by the projective representations and 1D representations of the symmetry group. The symmetric local unitary equivalence relation also allows us to obtain some simple results for quantum phases in higher dimensions when some symmetries are present.