Scaling Analysis in the Numerical Renormalization Group Study of the Sub-Ohmic Spin-Boson Model

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The spin-boson model has nontrivial quantum phase transitions in the sub-Ohmic regime. For the bath spectra exponent $0 \leqslant s < 1/2$, the bosonic numerical renormalization group (BNRG) study of the exponents β and δ are hampered by the boson state truncation which leads to artificial interacting exponents instead of the correct Gaussian ones. In this paper, guided by a mean-field calculation, we study the order parameter function $m(\tau = \alpha - \alpha_c, \epsilon, \Delta)$ using BNRG. Scaling analysis with respect to the boson state truncation N_b, the logarithmic discretization parameter Λ, and the tunneling strength Δ are carried out. Truncation-induced multiple-power behaviors are observed close to the critical point, with artificial values of β and δ. They cross over to classical behaviors with exponents $\beta = 1/2$ and $\delta = 3$ on the intermediate scales of τ and ϵ, respectively. We also find τ/Δ^{1-s} and ϵ/Δ scalings in the function $m(\tau, \epsilon, \Delta)$. The role of boson state truncation as a scaling variable in the BNRG result for $0 \leqslant s < 1/2$ is identified and its interplay with the logarithmic discretization revealed. Relevance to the validity of quantum-to-classical mapping in other impurity models is discussed.

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