From Ward Identity to Exact Transport Equation:
Complement to Éliashberg's Derivation of Landau-Silin Equation and beyond

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Transport equation of interacting fermions is re-considered in the light of the conservation law. We show that the exact equation for the particle-hole excitation is derived directly from the Ward identity by simply transforming it. Retaining the linear terms in the frequency, it is similar to that derived diagrammatically by Éliashberg[1], but takes two significant corrections along with it. The first is the dynamical wavefunction renormalization multiplying the frequency, which results in scaling other renormalizations. The second is the symmetrization of the collision integral with respect to the scatterings in and out.

We also discuss the problem of hidden ultraviolet divergence in Éliashberg's treatment of transport coefficients, thus claiming that the microscopic justification of Landau-Silin theory has not yet been completed. This divergence is shown to be removed by incorporating the particle-particle and hole-hole pairs of excitations into the above equation, without sacrificing its rigour. This requires re-identification of the distribution function, but the resultant transport equation for the new distribution function is identical, in its appearance, to the original one for the particle-hole pair.

[1]G.M. Éliashberg, Zh. Exp. Teor. Fiz. **41**, 1241 (1961); Sov. Phys. JETP **14**, 886 (1962).