# Cylindrical Free-Space Transmission-Line Metamaterials: A Numerical Study

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#### Abstract

This paper presents a new hollow cylindrical metamaterial composed of radially orientated negativerefractive-index transmission-line (NRI-TL) boards. The NRI-TL board orientation introduces radially inhomogeneous permeability and permittivity, whose profiles are extracted from full-wave simulations using a discrete multilayer approximation. Full wave simulations show that a vertical electric line source placed near a single metamaterial layer produces interesting radiation phenomena, including increased directivity and radiated power, as well as excitation of the dipolar resonant mode of the cylindrical shell.

### **1. Introduction**

Negative-refractive-index transmission-line (NRI-TL) metamaterials are composed of TL networks that are periodically loaded using reactive elements such that they exhibit a negative permittivity, negative permeability, and a negative refractive index (NRI) [1]. Volumetric NRI-TL metamaterials possessing these exotic qualities have been realized by stacking planar NRI-TL metamaterials. One example is the "Veselago-Pendry Superlens", which was shown experimentally to demonstrate subdiffraction imaging in free space at microwave frequencies [2]. In this work, we present full-wave finite-element-method simulations of a new cylindrical topology in which the same planar NRI-TL metamaterial boards are arranged radially, as shown in Fig. 1. This arrangement results in inhomogeneous material properties that vary with radius. The radial inhomogeneity is modelled discretely and HFSS is used to extract the effective medium parameters for each discrete layer. In this study, we consider an isolated cylindrical layer illuminated by a vertical electric line source placed within the simulation region. As the cylinder's inner radius is varied, we monitor both near fields and the radiation patterns, as well as the total radiated power.

## 2. Design & Simulation

The effective material parameters,  $\mu$  and  $\varepsilon$ , of stacked NRI-TL metamaterials depend on the spacing between the boards [2]. In a cylindrical arrangement, the spacing varies with radial position, rendering the effective medium radially inhomogeneous. To facilitate simulation, the cylindrical structure is discretized into a finite number of concentric cylinders, each of which possesses homogeneous properties approximated by a fixed spacing between adjacent boards. HFSS is then used to extract the material parameters for each discrete layer. The operating frequency for all simulations is chosen to be 2.40GHz.

The present work examines the single-layer arrangement shown in Fig. 2 (a), which consists of 12 radially orientated boards ( $\theta_b = 30^\circ$ ). The board spacing varies proportionally with the inner radius  $\rho_1$ , and decides the effective material parameters of the single layer. These parameters are extracted from HFSS simulation results of a metamaterial slab of single-cell thickness and infinite transverse dimensions for board spacings between 2mm and 11mm, according to the procedure described in [2, 3]. Figure 2 (b) shows the effective medium model of the single-layer structure.



Fig. 1: Cylindrical NRI-TL metamaterial with  $\theta_b$  being the angle between the boards.



Fig. 3: Extracted relative effective medium parameters for board spacings: a) 5.476mm, b) 7.476mm, c) 9.476mm.



Fig. 2: Single-layer cylindrical metamaterial with a vertical electric line source located 0.5mm from the outer radius: a) full-wave simulation model, b) effective medium model.



Fig. 4: Plot of power ratio as board spacing is swept.



Fig. 5: Complex electric field magnitudes and radiation patterns for increasing inner radii,  $\rho_1$ , (corresponding to effective relative permeabilities) of : a)7.01mm (-1.832-0.073*i*), b) 10.87mm (-4.079-0.323*i*), c) 14.74mm (-11.804-2.787*i*)

Although a metamaterial of single-cell thickness does not strictly satisfy the notion of an effective medium, we observe a close agreement between the effective parameters extracted for metamaterials of 1-, 3-, and 5-cell thicknesses in the effective-medium limit. As will be seen below, we also observe intriguing phenomena associated with homogeneous cylinders of the same thickness. Figure 3 shows the extracted effective material parameters as a function of board spacing of the cylindrical structure; that is, each board spacing corresponds to specific values of  $\mu$  and  $\varepsilon$ . It is worthy to note that  $\mu$  is slowly varying until the board spacing approaches 10.5mm, at which point a resonance in  $\mu$  occurs.

The cylindrical metamaterial is surrounded by a vacuum region with periodic boundary conditions on the top and bottom surface that render the structure infinite in the axial direction. The structure is excited with a vertical electric line source modeled using a thin PEC cylinder with a defined current on its surface. We fix the position of the source 1mm from the metamaterial's outer radius, and sweep the inner radius  $\rho_1$ . As the board spacing increases, the corresponding real part of the effective permeability becomes increasingly negative, while the imaginary part, modeling conductor, dielectric and lumped-component losses, increases.

Figure 5 shows the corresponding complex electric field magnitudes in the azimuth plane and radiation patterns, in which the 0-dB mark refers to directivity of the source in the absence of the metamaterial structure. In the cases shown, it is observed that we can obtain an increase in directivity of up to 3dB, as well as an improvement in front-to-back ratio of as much as 5dB. However, it is most instructive to examine the metamaterial's effect on the power ratio (PR), defined as the ratio of radiated power with the metamaterial in place to that with the line source alone. Figure 4 presents the PR versus board spacing. We observe a peak in PR when the board spacing is 7.476mm, corresponding to  $\mu \approx -4$ ,  $\rho_1 = 10.87$ mm and  $\rho_2 = 18.01$ mm. These values satisfy the dipolar resonance condition,  $\rho_1/\rho_2 \approx (\mu + \mu_0)/(\mu - \mu_0)$ , which has been observed to produce dramatic increases in PR for homogeneous single-layer cylindrical structures [4]. Indeed, an examination of the electric field distribution in Fig. 5 (b) suggests a dipolar-type resonance, in spite of the fact that the metamaterial is only a single cell in thickness. As previously mentioned, these are lossy structures and if losses can be reduced, we expect to see sharper and higher peaks in the PR curves.

#### 4. Conclusion

A cylindrical NRI-TL metamaterial structure composed of radially orientated NRI-TL boards, is shown to have radially inhomogeneous  $\mu$  and  $\varepsilon$  profiles that can be modelled in a discrete multilayer fashion. In this work, we presented full-wave simulations illustrating that cylindrical structures as little as one unit cell in thickness can improve the directivity, front-to-back ratio, and radiated power of a vertical electric line source. As observed for homogeneous cylindrical structures, maximum radiated power is obtained when the geometry of the structure satisfies the conditions for the excitation of the dipolar resonant mode.

#### References

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