Optimal power transfer by coupled resonant coils

L. Solymar and E. Shamonina

Optical and Semiconductor Devices Group, Electrical and Electronic Engineering (EEE) Department,
Imperial College, Exhibition Road, London SW7 2BT, UK,
e-mail: laszlo.solymar@hertford.ox.ac.uk, e.shamonina@imperial.ac.uk

Abstract

Assuming one transmitter and N-1 magnetically coupled receivers located in arbitrary positions a method is presented for finding the load impedances that optimize the total amount of power absorbed by the receivers. Auxiliary conditions are discussed.

1. Introduction

Ever since the times of Marconi we had to face the dichotomy: wired or wireless. They had their own territories: wired for telephones, wireless for broadcasting. Then microwave relays and satellite communications appeared presaging the demise of all things wired. The counter-offensive came with optical fibres which, after a relatively short time, replaced satellites for Trans-Atlantic communications. Optical fibres are here to stay. They are well entrenched whenever large amount of information needs to be transferred. On the other hand the wired world is rapidly losing the battle for voice communications. The tendency, irreversible it seems, is to change to mobile 'phones. However there is still one domain where the rule of wires is not much contested. It is transfer of power. Nobody in his right mind would want to have wireless transfer of power to an oven or a refrigerator but when it comes to the charging of laptops, or mobile devices in general, wireless transfer of power may become feasible as suggested by recent studies [1-3]. In this context it was shown by Sydoruk et al. [4] that transfer of power by Magnetoinductive waves boils down to the problem of matching. Whatever is the network it is always possible in a lossless system to transfer half of the incident power from a transmitter to a receiver. In the present paper we shall investigate the case when power needs to be transferred from a transmitter to a number of receivers at arbitrary locations. We shall use the techniques developed in the past for the optimal transfer of power by Magnetoinductive waves.

2. Mathematical formulation

We shall assume that there are N resonant coils, represented by series L,C circuits each one having a resistance R_e. The first one is a transmitter that is excited by a voltage V_1 from a generator with an internal resistance, R_1. There are N-1 further resonant coils arbitrarily located having the same L,C,R_e parameters, each one having a load impedance, $Z_{Ln}= R_{Ln} + jX_{Ln}$. In response to the applied voltage in coil 1 there will be currents flowing in each coil. In the corresponding $N \times N$ symmetric impedance matrix $Z_{nn} = Z_{Ln} + R_e + j(\omega L - 1/\omega C)$ are the self impedance and $Z_{mn} = jX_{mn}$ ($m \neq n$) is the mutual impedance. The relationship between applied voltage and current is given by the generalized Kirchhoff equation [see e.g. 4]

$$V = ZI , \tag{1}$$

where $V = [V_1, \ldots 0 \ldots 0]$ and $I = [I_1, \quad I_n \ldots I_N]$ are N-dimensional column matrices. The current may be obtained by inverting Z in Eq (1),

$$I = Z^{-1} V . \tag{2}$$

The power extracted from the nth coil is given as

$$P_n = \frac{1}{2} | I_n |^2 R_{\mathrm{L}n} \qquad \text{for} \qquad n = 2 \ldots N. \tag{3}$$

The variables are $R_{\mathrm{L}n}$ and $X_{\mathrm{L}n}$. They can be determined so as to optimise some criterion. One could for example choose the quite general criterion that

$$P_{\mathrm{total}} = \sum_{n=2}^{N} P_n = \mathrm{Max!} \tag{4}$$

subject to the auxiliary condition that

$$P_n = A_n P_{\mathrm{total}} \qquad \text{with} \qquad \sum_{n=2}^{N} A_n = 1, \tag{5}$$

where A_n gives the desired power distribution to be extracted by the coils. There may of course be many other type of auxiliary conditions requiring for example that the voltage in the nth coil exceeds some specified voltage. The solutions are likely to be obtained by numerical methods. For one particular case (no losses) we would like to make the following conjecture: It is always possible to transfer one half of the input power to N-1 properly matched resonant coils wherever they are placed.

3. Examples

We shall show a large number of examples at the Conference. In this Abstract we shall restrict generality and consider only two simple examples both at the resonant frequency.

(i) Two elements, one transmitter, one receiver. There is then a fairly simple analytical solution with optimisation for P_2. The condition to maximise P_2 yields for the load resistance,

$$R_{\mathrm{L}2} = R_{\mathrm{e}} + \frac{X_{12}^2}{R_{\mathrm{e}} + R_1}, \tag{6}$$

and the corresponding input and output powers are

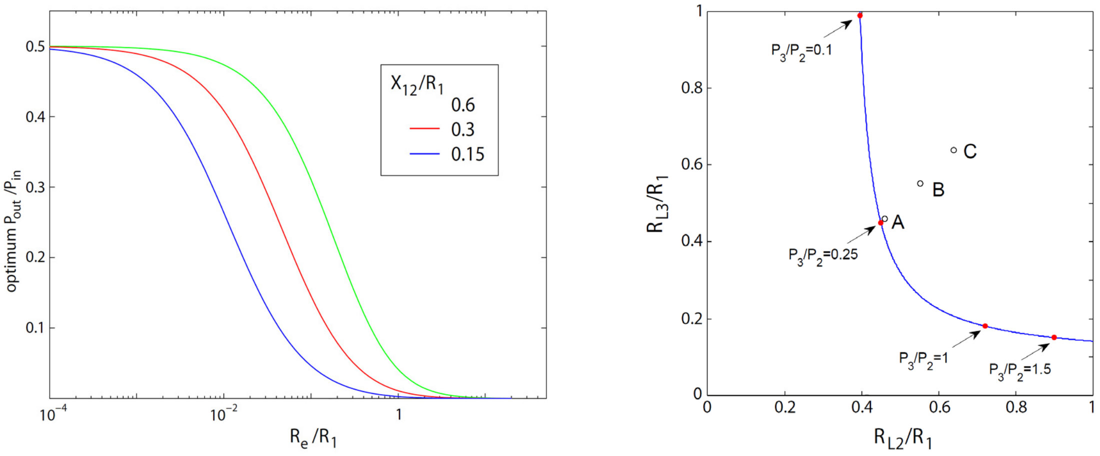
$$P_1 = \frac{V_1^2 R_1 (R_{\mathrm{e}} + R_{\mathrm{L}2})^2}{8 R_{\mathrm{L}2}^2 (R_{\mathrm{e}} + R_1)^2}, \qquad P_2 = \frac{V_1^2 X_{12}^2 R_1}{8 R_{\mathrm{L}2} (R_{\mathrm{e}} + R_1)^2} \tag{7}$$

We shall take $R_1 = 50$ ohm and $X_{12}/R_1 = 0.15$, 0.3 and 0.6. Note that we are not concerned here with the actual configuration of the two coils. Our main aim is to show how the optimum efficiency depends on losses. This is shown in Fig. 1a where the optimum value of $P_{\mathrm{out}}/P_{\mathrm{in}}$ is plotted as a function of R_{e}/R_1. As known [5] the optimum lossless efficiency is 50% and it is independent of the value of the mutual impedance between the elements. As losses increase the optimum efficiency is bound to decline and the decline depends on the value of the mutual impedance. The smaller is the mutual impedance the faster is the decline.

(ii) Three elements, one transmitter, two receivers. $R_1 = 50$ ohm again. The transmitter and the receivers are coupled to each other but not by an equal amount. We shall choose $X_{12}/R_1 = 0.6$ and $X_{13}/R_1 = 0.3$. It is further assumed that the mutual impedance between the two receivers can be neglected. Now the optimisation process is a little more complicated. First (see Fig. 1b) we find the curve in the $R_{\mathrm{L}2} - R_{\mathrm{L}3}$ plane which maximises the overall efficiency, $(P_2 + P_3)/P_{\text{in}}$. At various points in this curve the relative amount of power in the two receivers, P_3/P_2 are noted. In the range shown it can vary between the low value of 0.1 and the high value of 1.5. Note that it is possible to have higher power in element 3 than in element 2 in spite of the fact that the mutual impedance favours element 2 by a factor of 2. Next we can introduce losses. In that case there is no longer an optimum curve only optimum points (denoted by A, B and C) in the $R_{L2} - R_{L3}$ plane. At each of these points there are optimum values of R_{L2} and R_{L3}. Points A, B and C correspond to values of R_e/R_1 = 1/50, 1/5, 3/5 leading to efficiencies of 0.46, 0.24 and 0.18 respectively. A resistance of 1 ohm may be seen to reduce efficiency by not more than 8%.

4. Conclusions

An optimisation method using the generalised Kirchhoff equation is proposed for distributing power from one transmitter into receivers placed at arbitrary locations. Examples for two and three elements have been given showing the values of the load resistances optimising the output power and showing how the optimum efficiency declines as losses increase. The main conclusion appears to be that moderate losses have only a minor influence on efficiency.



(a) (b)

Fig. 1 (a) Single receiver: Variation of the optimum power transfer as a function of normalised loss for three different values of coupling. (b) Two receivers: The curve of maximum (=0.5) efficiency in the R_{L2}/R_1-R_{L3}/R_1 plane. Circles denote the maximum efficiency points for three different values of losses.

Acknowledgments: *ES gratefully acknowledges financial support of the Leverhulme Trust.*

References

[1] Kurs, A., Karalis, A., Moffatt, R., Joannopoulos, J. D., Fisher, and Soljacic, M. Wireless power transfer via strongly coupled magnetic resonances', Science, 317, 83-86, 2007.

[2] Cannon, B. L., Hoburg, J. F., Stancil, D., and Goldstein, S. C. 'Magnetic Resonant coupling as a potential means for wireless power transfer to multiple small receivers, IEEE Trans. Power El. 24, 1819-1825, 2009.

[3] Sample, A. P., Meyer, D. A. and Smith, J. R. Analysis, experimental results and range adaptation of magnetically coupled resonators for wireless power transfer, IEEE Trans. Ind. El. 58, 544-54, 2011

[4] Sydoruk, O., Shamonina, E. and Solymar, L. Tailoring of the subwavelength focus, Microw. & Opt. Technol. Lett., 49, 2228-2231, 2007