Noise Waves in Periodic Electrical Lattices

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Abstract

Passive electrical lattices can be arranged in a variety of configurations to allow forward and backward wave propagation. Considerable attention has been paid to propagation loss arising from the finite conductivity of inductive elements. Here we show that loss is inevitably accompanied by noise, that noise must also propagate as a wave, and demonstrate simple methods of calculating its power spectral density.

1. Introduction

Periodic electrical circuits have long been of interest to electrical engineers [1], and the development of metamaterials has attracted increasing interest for more general applications [2]. Many different configurations exist, and have been studied in one, two and three dimensions. For example, 1D L-C ladders (Fig. 1a) provide low-pass operation, magneto-inductive ladders (Fig. 1b) and electro-inductive ladders provide band-pass operation and C-L ladders (Fig. 1c) allow high-pass operation [1,3-5]. Depending on the dispersion relation, propagation may involve forward- or backward waves. A common feature is the presence of lossy inductors, which limit the Q-factor of resonators, introduce propagation loss and allow out-of-band propagation. An aspect that has largely been ignored is the Johnson noise that must arise in any circuit containing resistive elements [6, 7]. 'Noise waves' have been noted in distributed amplifier circuits [8]. As we show here, noise will also propagate as a wave in metamaterials, and propagation between elements will affect its power spectral density.

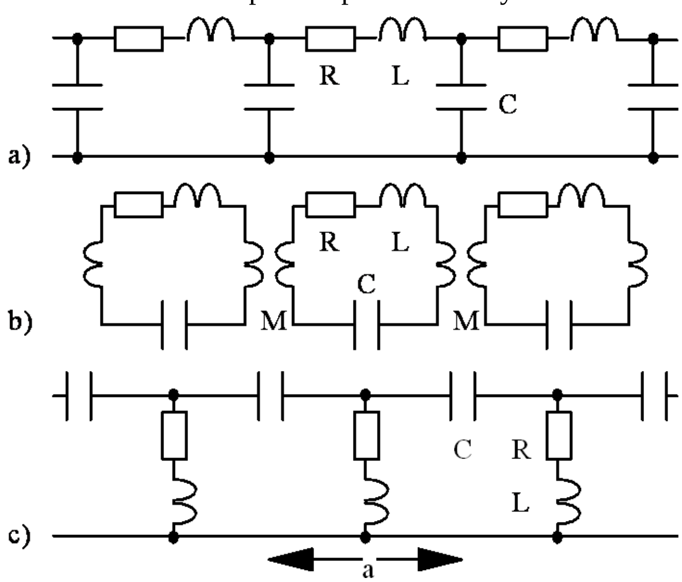

Figure 1. a) Low-pass, b) band-pass and c) high pass electrical lattices.

2. Noise waves

The effect of noise can be found as follows. Following [6] and [7], we assume that Johnson noise in any resistor R can be modelled as a voltage source of amplitude V_0, where the average value of $V_0V_0^*$ is $V_0V_0^* = 4kTRdf$, where k is Boltzmann's constant, T is absolute temperature and df is a small frequency interval. Noise arising from a single resistor in the series or shunt branch of element zero of a ladder can be found by placing a voltage source as shown in Figs. 2a and 2b, respectively. In each case, the effect is to excite waves that redistribute the noise power. For forward waves, the current in element n can be written as $I_n = I_0 \exp(-j|n|ka)$, where k is the propagation constant and a is the period. If k satisfies the dispersion relation of the undriven ladder, this solution satisfies the recurrence relation everywhere except in element zero, while the amplitude I_0 can be chosen to satisfy the driven equation at this point. Generally, loss will lead to a complex propagation constant $k = k' - jk''$. Consequently, the wave power will decay away from the source, and this can be expressed as the variation $I_n I_n^* = I_0 I_0^* \exp(-2|n|k''a)$.

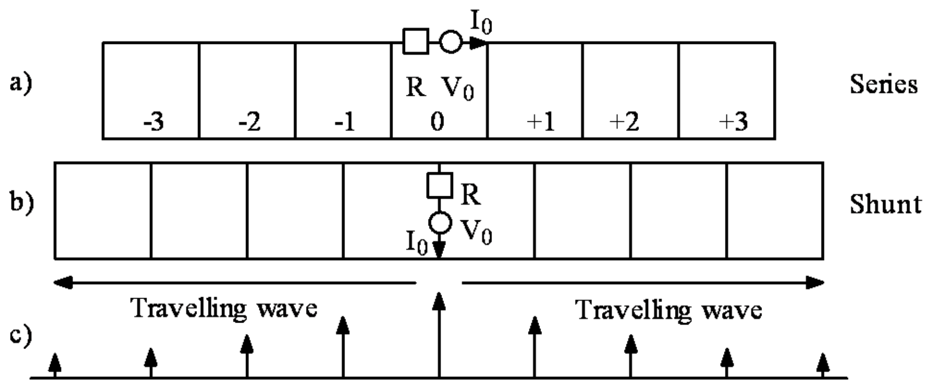

Figure 2. Effect of single noise voltage source.

More generally, a similar noise source will exist in every element, leading to a superimposed set of shifted excitation patterns as shown in Figure 3. Their effects may be summed by assuming that the noise voltages are uncorrelated. Writing $I_{n,n'}$ as the amplitude of the current wave at element n due to a noise source at element n', the total noise in element zero can be defined in terms of the summation $I_0 I_0^* = {}_{n'=-\infty}\Sigma^{\infty} I_{0,n'} I_{0,n'}^*$. The result is clearly a function of frequency, and can be compared with the flat distribution of Johnson noise to yield the normalised power spectral density resulting from incorporation of a set of noise sources in a lattice with additional reactive elements.

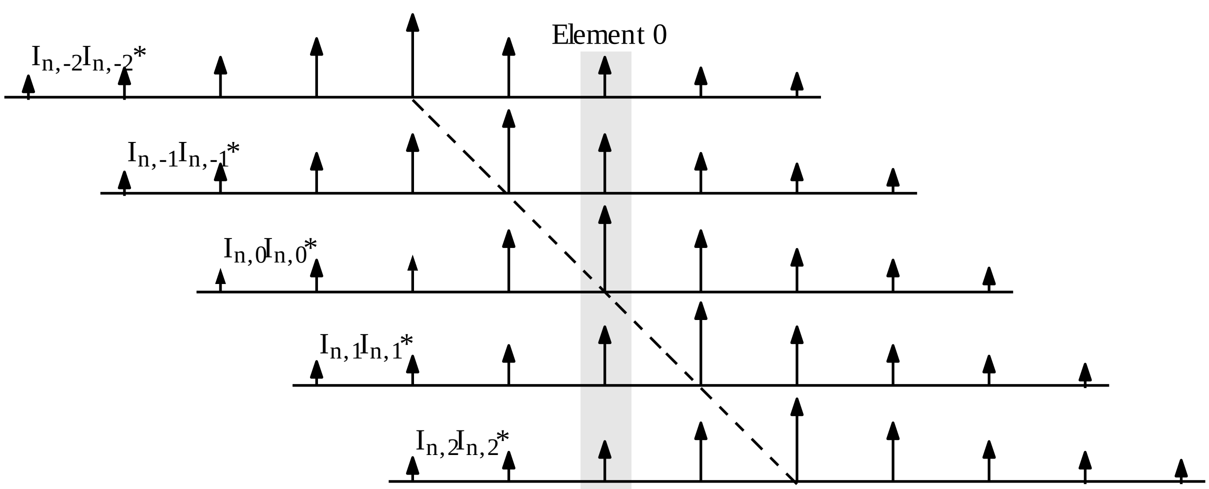

Figure 3. Noise power summation process.

3. Noise spectra

Example dispersion characteristics, attenuation characteristics and normalised noise power spectral densities are shown in Figs. 4a and 4b for low-pass (L-C) and band-pass (magneto-inductive) ladders. In each case, results are shown as a function of ω/ω_0, where $\omega_0^2 = 1/LC$. Low-pass results are calculated for a Q-factor of $Q_0 = \omega_0 L/R = 50$, and band-pass results for $Q_0 = 100$ and $\kappa = 2M/L = 0.2$. In each case, loss allows out-of-band propagation. The noise extends across the band, peaking at the edge where the dispersion characteristic is flat and decaying rapidly outside. Also shown in Fig. 4b is the noise PSD for an isolated resonant element. Similar approaches may be used to calculate the noise spectra in 2D and 3D lattices, and to find the noise figure when a lattice is used as a link.

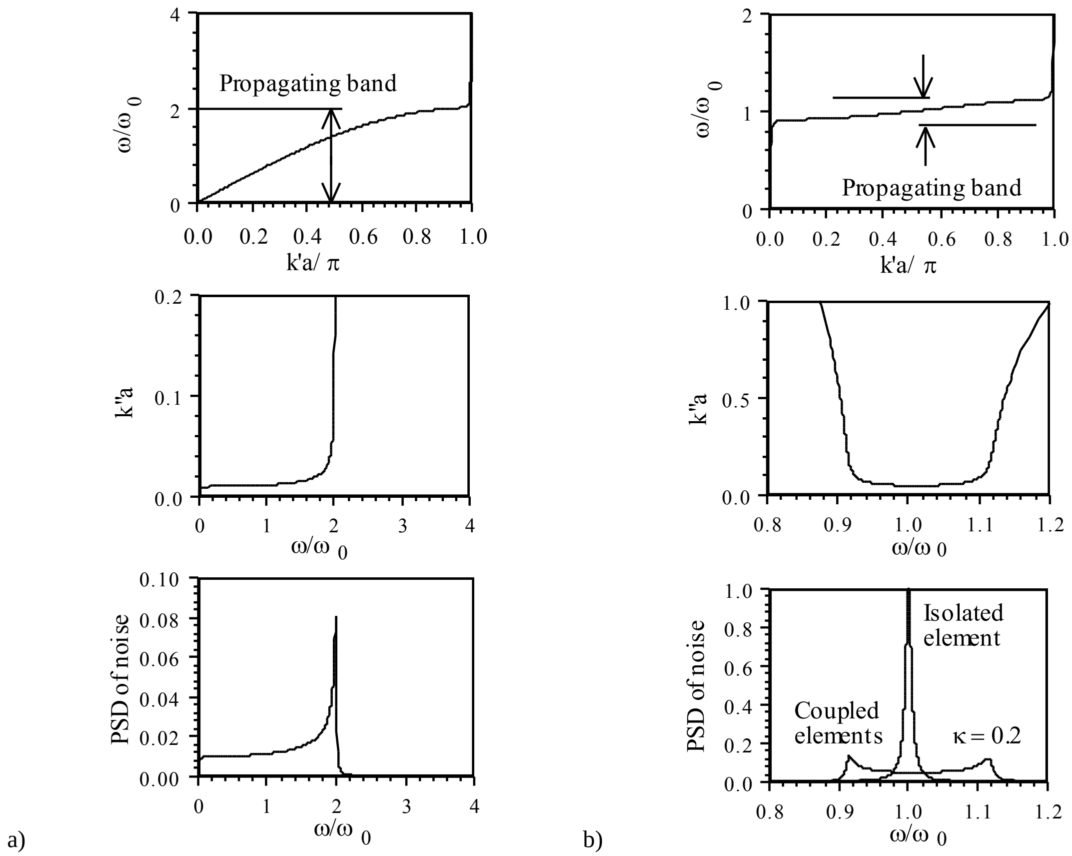

Figure 4. Dispersion, attenuation and noise characteristics of a) low-pass and b) band-pass lattices.

4. Conclusions

Noise is inherent in metamaterials containing conductive elements. Noise spectra for representative lattices show a distribution of noise across the entire propagating band. In contrast to loss, the effect of noise cannot be countered by amplification. Without careful design, the effect on practical applications is likely to be a poor noise figure in measurement systems.

References

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