Numerical MoM treatment of cloak with cyclic symmetry

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Abstract

A numerical analysis technique based on Array Scanning Method (ASM) for cyclic symmetry cylindrical cloaks is presented along with Method of Moments (MoM) for 1D periodic structures used for treating the periodicity along the axis of the cloak. The numerical analysis for a cloak presented in [8] is discussed.

1. Introduction

In the recent years several approaches to electromagnetically cloak an object have been documented in the literature. A succinct review of some of the methods has been provided in [1]. One of the earliest realization of a cylindrical cloak composed of radially placed resonant particles was presented in [2]. Along with that, some other cylindrical cloaks have been proposed in the literature that require the use of complex periodic structures. A fast method to analyze the scattering characteristics from the cloak exploiting its azimuthal symmetry and periodicity along the axis of the cloak can be very helpful.
An efficient numerical analysis technique based on the Method of Moments (MoM), coupled with the Array Scanning Method (ASM) is presented in Section 2 . In Section 3, simulation results for a volumetric cylindrical cloak composed of conical metal plates from [8] are presented.

2. Array Scanning Method

As shown by Sarkis et al.[5], the Array Scanning Method [3], often exploited in the framework of regular-array analysis [4], can also be exploited for the analysis of circular arrays. As mentioned in [5], the resulting method has a mathematical relationship with [7]. While the ASM provides approximations for the case of finite regular arrays, it allows the determination of exact solutions for circular structures. Those structures correspond to the rotation of N identical sectors of angular width $2\,\pi/N$. Compared to the direct solution of the whole structure, the savings in terms of calculation time are of the order of N^2. In the following, the method is first recalled for the case of a source limited to one sector. It is then extended to the case of an arbitrary excitation law, which may for instance correspond to a plane wave which -obviously- does not share the symmetry of the structure.

The Method-of-Moments system of equations may be written as $Z\,x = v$, where Z can be partionned in blocks corresponding to different sectors, while x and v are partitioned into segments. Below, indices will refer to different sectors. Let us first consider that only v_1 is non-zero. This type of excitation may be regarded as a superposition of periodic excitations:

$$v^{(1)} = \sum_{n=0}^{N-1} v_{(1)}(\phi = 2\,\pi\,n/N) \tag{1}$$

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with $$v_{(1)}(\phi) = \left[v_1^T\, e^{-j\,\phi\,0}, v_1^T\, e^{-j\,\phi\,1}, ..., v_1^T\, e^{-j\,\phi\,(N-1)}\right]^T \tag{2}$$
This result immediately stems form the values of the N complex roots of 1. $x_{(1)}(\phi)$, which satisfies $Z\,x_{(1)}(\phi) = v_{(1)}(\phi)$, has the same symmetry and its value can be obtained from the first block row of the initial system of equations. The other rows produce the same result. Exploiting the circular symmetry of the problem, the pertaining equation reads:
$$\left(\sum_{n=0}^{n-1} Z_{1n}\, e^{-j\,n\,\phi}\right)\, x_{(1),1}(\phi) = v_1 \tag{3}$$
The corresponding solution in sectors other than sector 1 is obtained by simply applying the proper phase shift. The solution to the excitation described in (1) is simply obtained by an analogous superposition of solutions:
$$x^{(1)} = \sum_{n=0}^{N-1} x_{(1)}(\phi = 2\,\pi\,n/N) \tag{4}$$
This result can be extended to an arbitrary excitation over the whole structure by superimposing N excitations, originating from the different excitations found in the different sectors. For instance, the contribution of v_2 to the excitation becomes:
$$v_{(2)}(\phi) = \left[v_2^T\, e^{j\,\phi\,1}, v_2^T\, e^{j\,\phi\,0}, ..., v_2^T\, e^{-j\,\phi\,(N-2)}\right]^T \tag{5}$$
One should note that compared to (2), a phase shift has been applied, such that the excitation remains unchanged in sector 2 instead of sector 1. As was the case for excitation in one sector only, the solution for excitations over the whole structure $x_{(a)}$, restricted to the first sector, is obtained from the first block-row system of equations. The solution over all sectors is obtained by periodizing the solutions while applying the proper phase shifts and then a summation analogous to (4) is used to superimpose the solutions ($x_{(1)}$ replaced by $x_{(a)}$, $x^{(1)}$ by x). Along the axis of the cloak, the structure is periodic with period a and inter-cell phase shift equal to ψ. For staight-on excitation of the cloak, we will generally consider $\psi = 0$). This problem is solved with the Method-of-Moments while exploiting the 1D periodic scalar Green's functions and the mixed-potential integral equation is written. The details of the formulation can be found in [6].
$$G(R, x) = \frac{1}{4\,j\,a} \sum_{n=1}^{N} H_0^{(2)}(k_{\rho,p}\,R)\, e^{-j\,k_p\,x} \tag{6}$$
with $k_{\rho,p}^2 + k_p^2 = k^2$ and $k_p = \psi/a + p\,2\,\pi/a$. R is the straight distance from the cloak axis.

3. Numerical Simulation

The cloak proposed in [8] has been studied using the numerical analysis technique discussed above. Fig.1 shows the mesh of a periodic sector created using GMSH with the dimensions provided in [8].

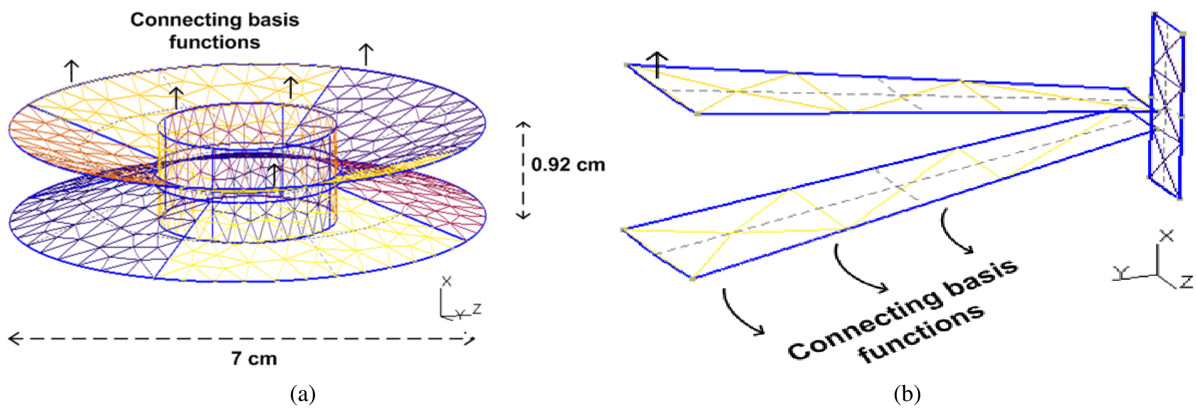
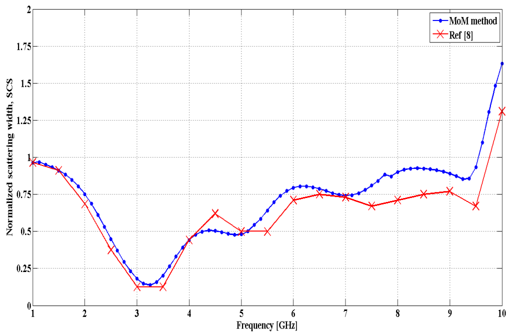


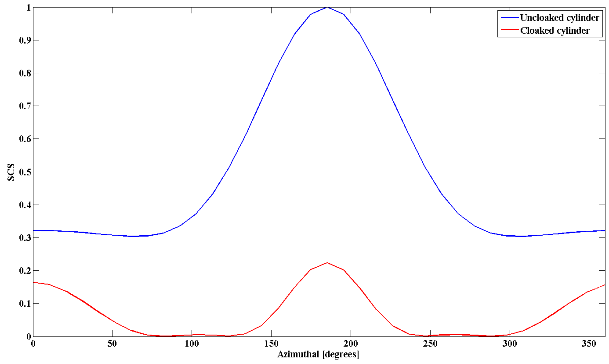
Fig. 1: Mesh of the cloak (a) full cloak with cylinder (b) a sector of the cloak and cylinder

The azimuthal sector is $\pi/18$ in the simulated example. Basis functions overlapping consecutive sectors are used to ensure current flow between sectors. The calculation time is reduced by the order of N^2, where N is the number of azimuthal sectors. An order N is saved in terms of memory. The cloak is periodically repeated along the x axis.The periodicty is the same as the opening of the tapered waveguide at the outer radius of the cloak. Therefore connecting basis functions are imposed even along the cylinder axis to allow the continuity of the current flow as indicated in Fig. 1.

The SCS of the cloaked cylinder normalized to the uncloaked cylinder is computed over a frequency band from 1 to 10 GHz and is shown in Fig. 2a. The results are in good agreement over the cloaking band, and slightly differ at higher frequencies. The SCS is found to be minimum at 3.25 GHz where the scattering cross section of a perfectly conducting cylinder is reduced by almost 90% as shown in Fig.2b. Only the polarization of the incident field parallel to the axis of the cylinder has been simulated here to compare with the results presented in [8].



(a)



(b)

Fig. 2: (a) SCS of the cloaked cylinder normalized to the uncloaked cylinder over frequency compared to [8] (b) SCS of the cloaked and uncloaked cylinder at 3.25 GHz over azimuthal.

4. Conclusion

A fast and efficient numerical technique for analysis of scattering characteristics of a cylindrical cloak periodic along the axis of the cylinder and having cyclic symmetry along the azimuthal has been presented. To exploit the cyclic symmetry, the Array Scanning Method is used, and, for the periodicity along the axis of the cylinder, the periodic Method of Moments is used. An order N saving is obtained in terms of memory whereas the solution time is reduced by N^2 where N is the number of cyclic sectors. The scattering cross section of a cloaked cylinder normalized to an uncloaked cylinder over frequency is shown and the results are found to be in agreement to those published in [8]. Currently, this analysis technique is being extended to study new cloak designs with reduced computation time and increased efficiency.

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