Resonant multiple scattering by metamaterial clusters

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Abstract

Metamaterials are composed of very strongly scattering subwavelength objects. Hence, metamaterial scatterers in engineered subwavelength configurations can give rise to interesting magnetic analogons of plasmonic antennas, plasmon hybridization, extraordinary transmission, etcetera. In this paper we discuss new insights in such phenomena obtained from a new fully electrodynamic theory for magneto-electric point scatterers. This theory generalizes all common magneto-static circuit models for metamaterials by adding retardation, interference and energy conservation. Uniquely, our theory provides quantitative cross sections, resonance linewidths, and reflection and transmission coefficients for arbitrary clusters of objects like split rings. We confirm these quantitative predictions in scattering experiments, and predict important fundamental constraints.

1. Introduction

In addition to their surprising interaction with the magnetic field of light, metamaterials distinguish themselves in one fundamental aspect from ordinary materials. While the permeability and permittivity in ordinary materials derives from very large densities $\rho \gg \lambda^3$ of weakly polarizable (polarizability $\alpha \ll \lambda^3$) objects to achieve an appreciable value of, e.g., $\epsilon \sim 1 + \rho\alpha$, metamaterials are typified by low densities of building blocks which have a large polarizability per building block. This property makes the building blocks interesting for many applications outside metamaterials, similar to current interest in plasmonics scatterers for e.g, waveguides, antennas, quantum plasmonics and field-enhanced spectroscopy. The strength of scattering as gauged by polarizability (units of volume) or cross section

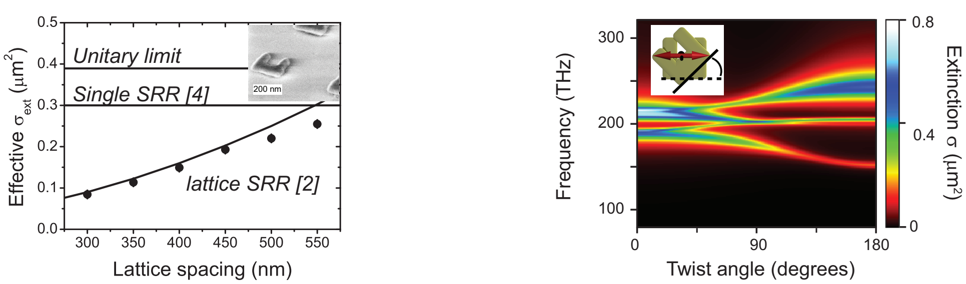

Fig. 1: (Left panel) Data points: the *effective* extinction cross section per split ring in cubic arrays (black dots) is as large as the array unit cell area (curve [2]), until it reaches the single split ring value [4] for very dilute arrays, which is close to the unitary limit. These observations for split rings resonant at 1.4 μm (SEM micrograph, angled view) imply that metamaterial arrays have strong multiple scattering interactions. (Right panel) Extinction cross section predicted for lossless twisted split ring dimers [6].

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is not just large compared to the geometrical size of the metamaterial scatterers. Scattering strength can also be benchmarked against a universal 'unitary limit' that only depends on the host wavelength λ, and which states that for any dipolar scatterer the extinction cross section is always limited to less than $\sigma_{\text{ext}} = 3/2\pi\lambda^2$ (equivalently $|\alpha| \leq 3/2(\lambda/2\pi)^3$) [1]. Metamaterial building blocks are close to this limit, and metamaterials might hence be most appropriately viewed as strongly multiply scattering systems. Recently, such ideas have come to the fore in several experiments that reveal strong interactions between split rings. For instance, recently measured transmission spectra of lattices of split rings demonstrate large variations in resonance frequency and width as a function of lattice geometry and density [2]. Extinction measurements on single split rings also confirm that cross sections are close to the unitary limit [4]. Finally, experiments on, e.g., stereometamaterials [3], and chirality due to single building blocks [5] or the stacking thereof also hinge on strong coupling effects. In this contribution we discuss many new insights offered by the simplest possible electrodynamic model that generalizes existing electrostatic and magneto-static circuit models, yet self-consistently deals with interference and retardation effects, to all scattering orders [6]. In addition we present experiments to explore these insights.

2. Sketch of a simple model

Simple circuit models are ubiquitous throughout the metamaterial community. Essentially, the collective resonances of of interacting building block are captured by coupled equations of motion for the current and charge in each building block, where mutual driving terms couple the resonances. For instance, for a set of N building blocks with capacitance C, inductance L, one solves for the resonances of [3]:

$$L\frac{dI_i}{dt} + \frac{Q_i}{C} = \sum_{j\neq i}\left[\frac{\kappa_E}{d_{i,j}^3}Q_j + \frac{\kappa_H}{d_{i,j}^3}\frac{dI_j}{dt}\right] \qquad i = 1\dots N \tag{1}$$

Here $\kappa_E/d_{i,j}^3$ and $\kappa_H/d_{i,j}^3$ quantify mutual electrostatic and inductive dipole-dipole coupling between elements at distances $d_{i,j}$ from each other. Such a formalism readily gives resonance frequencies, but contains no interference and yields no observables like resonance strengths, linewidths, cross sections, or radiation patterns. Instead, a typical scattering theory would be based on a self-consistent equation for the induced dipole moments $\mathbf{p}$ and $\mathbf{m}$ set up in response to some incident field $(\mathbf{E}_{\text{incident}}, \mathbf{H}_{\text{incident}})$:

$$\begin{pmatrix}\mathbf{p}\\ \mathbf{m}\end{pmatrix} = \overleftrightarrow{\boldsymbol{\alpha}}\left[\begin{pmatrix}\mathbf{E}_{\text{incident}}\\ \mathbf{H}_{\text{incident}}\end{pmatrix} + \sum_{j\neq i}\overleftrightarrow{\mathbf{G}}(\mathbf{r}_j,\mathbf{r}_i)\begin{pmatrix}\mathbf{p}_j\\ \mathbf{m}_j\end{pmatrix}\right] \qquad \text{with} \qquad \overleftrightarrow{\boldsymbol{\alpha}} = \begin{pmatrix}\alpha_E & \alpha_{EH}\\ \alpha_{HE} & \alpha_H\end{pmatrix} \tag{2}$$

where $\overleftrightarrow{\mathbf{G}}$ is the 6×6 electrodynamic dyadic Green function of free space, and $\overleftrightarrow{\boldsymbol{\alpha}}$ is a generalized 6×6 polarizability. This generalized polarizability, and constraints on its form set by symmetry and reciprocity (i.e., Onsager constraints), are well known in *electrostatic* analyses of bianisotropic particles, where it is understood that the off diagonal elements $\alpha_{EH} = -\alpha_{EH}^T$ correspond to the electric (magnetic) driving of magnetic (electric) responses, i.e., to bianisotropy that is ubiquitous in mematerials. Indeed, textbooks on bianisotropic materials [7] explain how to convert any circuit model, whether for Ω-particles, split rings, wire helices or other shapes, in a magneto-electrostatic polarizability tensor. However, any actual physical scatterer has radiation damping, meaning that the static polarizabilities violate energy conservation unless corrected with dynamic terms involving c. We recently proved a generalized Radiation Damping Addition theorem that allows to convert any magneto-electro-static polarizability into an energy conserving one

$$\text{inv}\,\overleftrightarrow{\boldsymbol{\alpha}}_{\text{dynamic}}) = \text{inv}(\overleftrightarrow{\boldsymbol{\alpha}}_{\text{static}}) - \frac{2}{3}\frac{\omega^3}{c^3}\mathbb{I} \tag{3}$$

The above ingredients provide a powerful unification of (A) magneto-electrostatic models that deal with completely general magneto-electric responses but violate energy conservation, and (B) purely electric or uniaxial dipole models that contain c and energy conservation, but not magnetoelectric effects.

3. Predictions for experiments

The repercussions of this unified model are far reaching. An important insight is that fundamental properties are best understood not through the response under 'Cartesian' incidence conditions often chosen in experiments, but that they can best be understood by finding the eigenvalues and vectors of the 6×6 polarizability. Some findings relevant for the design of metamaterial scatterers are the following:

- The strongest magneto-electric effect is limited to $|\alpha_{EH}| \leq \sqrt{\alpha_E \alpha_H}$ in planar scatterers.
- Any planar scatterer described by a simple circuit model operates exactly at this limit of strongest possible magneto-electric response $|\alpha_{EH}| = \sqrt{\alpha_E \alpha_H}$.
- Any magneto-electric scatterer has chiral eigenmodes. Hence structural chirality must be ubiquitous in the extinction cross section of planar, strongly magnetic metamaterial scatterers, and bi-anisotropy is almost unavoidable in planar metamaterials.
- Radiation patterns (i.e., differential scattering cross sections) of single magneto-electric metamaterial scatterers are very non-dipolar, even within this dipole model.

Our implementation of the multiple scattering model (2) for simple clusters, as well as infinite lattices (using 6×6 dyadic lattice sum calculations) of magneto-electric scatterers allows us to assess many important problems, such as the utility of magneto-electric diffractive gratings, the non-locality of effective medium parameters in actual arrays, the band structure of super-radiant and subradiant modes that will determine how the lasing spaser spases, or the use of split ring analogons of Yagi Uda antennas to control single emitters. As a simple example of the power of this model, Fig. 1 shows the quantitative extinction cross section versus wavelength that the model predicts for the 'stereo-metamaterial' experiment by Liu et al. [3], in which one split ring is stacked with some variable twist angle over a second split ring. Without any adjustable parameters, this calculation reproduces all features of experiment and FDTD simulations, including the difference in line-width between the two dominant resonances (super- and subradiant), the asymmetry in the magnitude of the splitting at twist angle 0 and 180 degrees, and the fact that an anti-crossing occurs, that is not centered at 90 degrees. Interestingly, all these features occur without any inclusion of, e.g., multipole corrections that the electrostatic model needs on an ad hoc basis to qualitatively match full-wave calculations. While multipole effects undoubtedly occur in experiments, it is evident that electrodynamic effects are very important for any fair comparison between experiments and dipolar coupling models, and that scattering in magneto-electric dipolar systems is much richer than hitherto assumed. At the conference I will also present current experimental efforts in our group to chart and apply these new insights. For instance, I will report on the first angle-resolved measurements of light scattered by individual nano-scale building blocks, that we measure using a Fourier microscope to verify our prediction that radiation patterns of single nano-objects encode the polarizability tensor.

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