Four different sign combinations of density and modulus exhibited by a single metamaterial

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Abstract

Possibility of negative values of permittivity and permeability discussed by Veselago [1] became reality by man-made structures consisting of subwavelength-size cells [2, 3]. New phenomena and applications began to come out as the materials properties were extended into the single and double negative quadrants of the ϵ-μ diagram [4, 5, 6, 7]. Here, we present a dispersive new acoustic metamaterial consisting of Helmholtz resonators and membranes, which exhibited all the four sign combinations of the constitutive parameters; namely density negative (ρNG), modulus negative(BNG), double negative (DNG), and double positive (DPS). Consequently, this acoustic metamaterial exhibits some important extreme properties including infinite stiffness and zero density.

1. Introduction

Smith *et al.* [4] reported construction of a composite medium consisting of interspaced split-ring resonators and wires, which became the key model for the subsequently developed variety of double negative metamaterials. However, the acoustic equivalence of this electromagnetic metamaterial has, so far, not been reported. Very recently, acoustic double negative medium with a composite structure was realized, but the dispersion relation does not exactly match with that of [4]. Here, we present construction of a composite structure consisting of an array of interspaced Helmholtz resonators and tight membranes, which is the acoustic counterpart of the electromagnetic metamaterial constructed by Smith *et al.* [4].

2. Negative Bulk Modulus and Density

If the Helmholtz resonator array were absent in the structure 1, it would be a simple tube with an array of membranes, which is identical to the density negative structure reported in [9]. Likewise, if the membranes in the present structure were removed it would be rendered to the modulus negative structure consisting of an array of Helmholtz resonators [10], which exhibits the effective modulus $B_{eff}(\omega)$. The density negative structure consisting of an array of membranes was reported to convey collective acoustic oscillation similar to the plasma oscillation.

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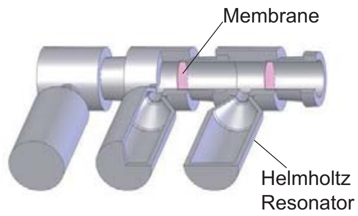

Fig. 1: Composite structure consisting of an array of interspaced membrane and Helmholtz resonators.

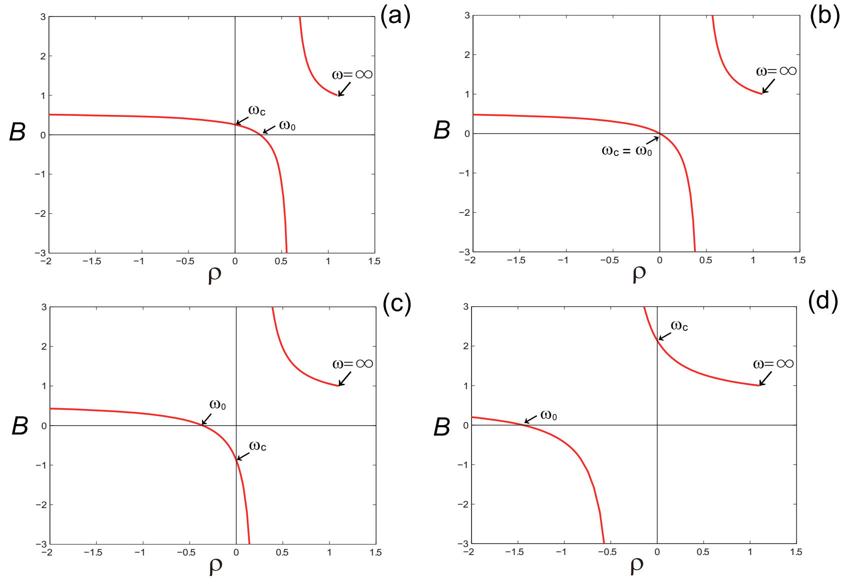

Fig. 2: (a)Graphs of states for the cases $\omega_c < \omega_0 < \omega_b$ (b)$\omega_0 = \omega_c < \omega_b$ (c)$\omega_0 < \omega_c < \omega_b$ (d)$\omega_0 < \omega_b < \omega_c$

As in the cases of previous works [4, 11], Negative bulk modulus and density simultaneously hold in the composite system. From these two materials one obtains the expression for the wave-vector, $k(\omega) = s\omega\sqrt{\rho_{eff}B_{eff}^{-1}}$, where s is the sign function which has values -1 for the frequencies in the double negative range $\omega_0 < \omega < \omega_c$, and +1 in the double positive region $\omega_h < \omega$. Explicitly,

$$k(\omega) = s\sqrt{\frac{\rho'(\omega^2 - \omega_c^2)(\omega^2 - \omega_h^2)}{B_0(\omega^2 - \omega_0^2)}}. \qquad (1)$$

3. ρ - B plane

Unlike the conventional media, the density and the modulus of the present metamaterial both change with the frequency. Such change corresponds to the motion of the (ρ, B) point on the $\rho - B$ plane. The trajectories of the motions are shown for the four possible cases in Fig. 2.

In the case of Fig. 2a, the curve intersects the ρ-axis and the B-axis. The intersecting points, $(\rho = 0, B > 0)$ and $(\rho > 0, B = 0)$, correspond to the states with the indexes $n = 0$ and $n = \infty$, respectively. The states in the line connecting these two points in the first quadrant have the index varying continuously from zero to infinity. Such property has potential impact for cloaking or transformation of waves.

In case the frequency ω_c lies in the middle of the frequencies ω_0 and ω_h, the metamaterial exhibits the

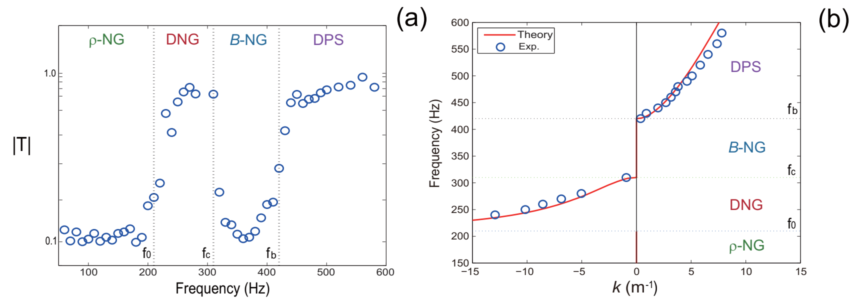

Fig. 3: Experimental results for the composite HR-membrane-structure. (a) Transmission data. (b) Wave-vectors.

pattern shown in Fig 2c, where the trajectory passes through all the four quadrants: From the trajectory one can see that the metamaterial is density negative for all the frequencies lower than ω_0, and in the range $\omega_0 < \omega < \omega_c$, this acoustic medium is double negative. It becomes, in the frequency range $\omega_c < \omega < \omega_h$, modulus negative. Above ω_h it turns double positive.

Experimental transmission measurement revealed two stop bands, as shown in Fig. 3a. These stop bands correspond to the frequency ranges $\omega < \omega_0$ and $\omega_c < \omega < \omega_h$ where the wave-vector in Eq. 4 becomes imaginary. For the pass bands, the experimental data were obtained by measuring the phase velocity v_{ph}, using the relation, $k = \omega/v_{ph}$, Fig. 3b. The data agree excellently with the theoretical curve from Eq. 4.

4. Conclusion

We presented fabrication of a double negative acoustic metamaterial, which is an acoustic version of the electromagnetic metamaterial consisting of split-ring resonators and continuous wires. This new acoustic metamaterial exhibited sharp transitions in wave characteristics at the three frequencies ω_0, ω_c, and ω_h, transforming from ρNG to DNG, BNG, and DPS states in sequence with increasing frequency. Also, in the DNG band, we observed index of refraction continuously changing two orders of magnetitude, which has potential use for transformation of acoustic waves.

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