A length-independent method to retrieve the effective parameters of materials and determine the interactions in metamaterial-structures

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Abstract

A novel method to retrieve the effective electromagnetic parameters (permittivity and permeability) is proposed. The advantage over the existing methods is that the length of the material under test (MUT) should not be strictly smaller than the wavelength. This makes it possible to retrieve the effective electromagnetic parameters of a metamaterial consisting of any number of cells. Consequently the effect of the interactions between the cells in a long structure can be studied without length-disturbance. The method is determined and its length-independency is proved. The interactions between the cells of a prototype with different lengths are discussed and the deviation of their effective electromagnetic parameters, which is caused by the interactions, is presented.

1. Introduction

Metamaterials for the microwave range consist of the artificial electromagnetic cells such as SRR and/or TW, they determine the effective electromagnetic parameters of the whole structure. A general method to retrieve the effective electromagnetic parameters of the unit cell (UC) of a metamaterial is by calculating $n = \sqrt{\epsilon \cdot \mu}$ and $z = \sqrt{\frac{\mu}{\epsilon}}$ from the measured or simulated S parameters [1]. The ϵ and μ are then directly calculated from $\mu = nz$ and $\epsilon = n/z$. Generally the complex values should be used:

$$\underline{n} = n' - jn'' \qquad \underline{z} = z' - jz'' \qquad \underline{\epsilon} = \epsilon' - j\epsilon'' \qquad \underline{\mu} = \mu' - j\mu'' \tag{1}$$

It is already known how the $\underline{n}$ and $\underline{z}$ can be calculated from complex $\underline{S}$ parameters [1, 2]. But here the same critical problems such as renormalization and unwrapping occur, which are discussed in this paper.

2. Retrieval of the effective $\underline{\epsilon}$ and $\underline{\mu}$

2.1. Retrieval Method

A cuboidal material is defined in „Computer Simulation Technology“(CST) Program. As incident wave the plane wave is used. The n' and n'' are calculated from the de-embedded S parameters as follows:

$$\underline{S}_{21} = e^{-\gamma L} = e^{-\alpha L - j\beta L} \qquad \Rightarrow \qquad |\underline{S}_{21}| = e^{-\alpha L}, \angle \underline{S}_{21} = -\beta L \tag{2}$$

In which L denotes the length of MUT, $\gamma = \alpha + j\beta$ is the propagation constant and α and β are the attenuation and the phase constants, respectively. On the other hand, assuming a complex $\underline{n}$ the

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attenuation and the phase constants can be explained with the complex $\underline{n}$ and the γ can be defined generally as follows:

$$\gamma = jk = j\underline{n}k_0 = j\underline{n}\frac{2\pi}{\lambda_0} = j(n' - jn'')\frac{2\pi}{\lambda_0} \quad \Rightarrow \quad \alpha = n''\frac{2\pi}{\lambda_0}, \beta = n'\frac{2\pi}{\lambda_0} \tag{3}$$

whereas k and k_0 are the guided and the free space wave numbers, respectively. Comparing equations 2 and 3, the n' and n'' can be determined as follows:

$$n' = -\frac{\angle \underline{S}_{21}\lambda_0}{2\pi L} \qquad n'' = -\frac{ln(|\underline{S}_{21}|)\lambda_0}{2\pi L} \tag{4}$$

$\underline{z}$ is retrieved form the reflection factor of the incident plane [3]. The solution of equation 6 determines $\underline{z}$. The $\underline{S}_{11}$ must not be renormalized and it's normalized impedance must be the free space impedance (See Fig. 1).

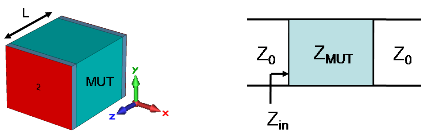

Fig. 1: Calculation of $\underline{z}$.

$$\underline{S}_{11} = \frac{\underline{Z}_{in} - Z_0}{\underline{Z}_{in} + Z_0} \tag{5}$$

with

$$\underline{Z}_{in} = Z_0 \frac{1 + j\frac{\underline{Z}_{MUT}}{Z_0}tan(\underline{\beta}L)}{1 + j\frac{Z_0}{\underline{Z}_{MUT}}tan(\underline{\beta}L)} \quad , \quad \underline{Z}_{MUT} = \underline{z}Z_0 \quad \Rightarrow \quad \underline{S}_{11} = \frac{\underline{z}^2 - 1}{\underline{z}^2 - \underline{M}\underline{z} + 1} \tag{6}$$

with $\underline{M} = 2jcot(\underline{\beta}L)$.

2.2. Renormalization and Unwrapping Problems

The measured or simulated $\underline{S}$ parameters are normally normalized with respect to the free space impedance. But the UC is usually inhomogeneous, which causes a changeable effective $\underline{z}$. To solve this problem the correct effective renormalized impedance is determined iteratively.

Another problem is the calculation of n'. The correct phase shift between the incoming and the outgoing waves is here required (Equ. 4). Unwrapping doesn't help here; since depends on the start-frequency of the simulation/measurement. This is due to the fact that the effective length of the MUT is unknown and the number of jumps of $\angle \underline{S}_{21}$ at frequencies lower than the start-frequency are then unknown (see Fig. 2 left). This means that k by the factor $\angle \underline{S}_{21} - 2k\pi$ is unknown. In Fig. 2 right n' for $k = 0, 1, 2, 3$ for a material with $\epsilon = 2.33, \mu = 1$ and $L = 50mm$ (a long material) is calculated. As shown only for $k = 2$ the correct n' is achieved. To solve this problem the gradient of n' is used. The idea is that the n' by the correct selection of k is at the non-resonant frequencies constant or its gradient is much lower than the gradient of n' caused from false k. As shown in Fig. 2 right the correct $k(k = 2)$ occurs where the $\nabla(n') \rightarrow 0$. This solution makes this method length-independent. Since the k is chosen from the $\nabla(n')$ and not from the start-frequency and the length of the model. This makes it possible to determine the correct interactions between the cells in a long structure and their effect on the effective $\underline{\epsilon}$ and $\underline{\mu}$ of the whole structure, which is shown for the prototype model in Fig. 3 left. Fig. 3 right shows the calculated parameters of these models. As shown the interactions of the models are clear to see even for the long model and the lengths of the models do not disturb the calculations.

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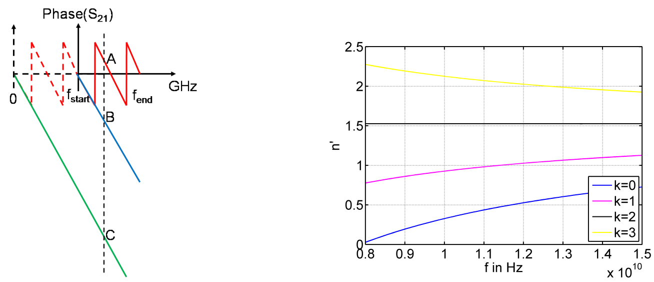

Fig. 2: The unwrapping depends on start-frequency (Left): A: Wrapped angle. B: Unwrapped angle. C: Correct angle. (Right): n' for k = 0,1,2 and 3.

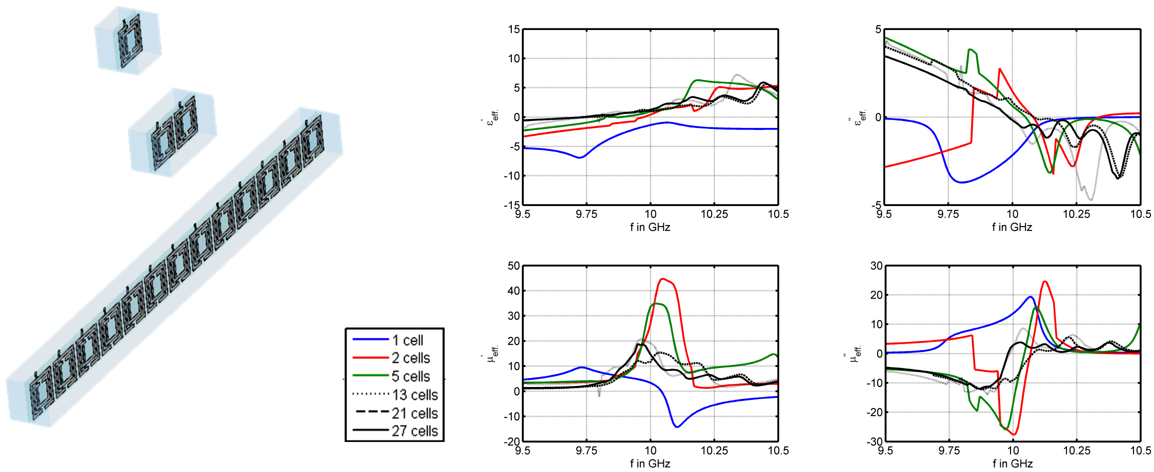

Fig. 3: The length-independent method makes it possible to study the Interactions between the cells. (Left): 1,2 and 13 cells metamaterial models used to prove the method. (Right): Interactions between cells for different lengths compared with the UC.

As also shown in Fig. 3 the calculated effective parameters converge in each case ($\epsilon', \epsilon'', \mu', \mu''$) by increasing the number of cells. This represents ,as expected, higher homogeneity of the metamaterial by increasing the number of cells.

4. Conclusion

A length-independent method to retrieve the electromagnetic parameters of materials is proposed. It's advantage is that the gradient of n' is used to find the correct phase. This makes this method, compared to the methods using „unwrapping, "length-independent. Consequently it is now possible to determine the interactions between any number of metamaterial cells particularly long ones.

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