On the accuracy of Surface Integral Equation Formulations for Left-Handed Materials

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Abstract

Several left-handed material (LHM) spheres with different constitutive parameters are analyzed employing different integral-equation formulations based on the Method of Moments. The combined normal formulation (CNF), the combined tangential formulation (CTF), the Poggio-Miller-Chang-Harrington-Wu-Tsai formulation (PMCHWT) and the electric and magnetic current combined field integral equation (JMCFIE) are tested in order to assess their accuracy and suitability for dealing with LHM's. The results point out that the JMCFIE formulation is the most stable and reliable proposal.

1. Introduction

Left-handed materials (LHM's) were already theoretically proposed in 1968 [1], though their negative refraction was not experimentally demonstrated until 2000 [2]. Many theoretical and experimental studies focused on these artificial structures have been developed in last years due to their wide range of applications and potential capabilities [3].

The electromagnetic numerical analysis of LHM's has been usually tackled by means of differential-equation formulations [4],[5] which require discretization of the object and the surrounding space. This fact limits the applicability of these techniques to realistic large problems. Recent works have shown that the application of surface integral equations (SIE), which have been extensively used for solving scattering problems involving homogeneous or piecewise homogeneous dielectric objects [6]-[9], may be extended to the homogeneous LHM's analysis [10]-[12]. In this work, the well-known Method of Moments (MoM) SIE formulation [13] is applied to predict the electromagnetic scattering of LHM's.

2. Integral Equation Formulations

For homogeneous dielectric materials, it is usual to consider the combination of normal and tangential equations derived from the boundary conditions imposed separately to the electric and magnetic fields, namely, the tangential/normal electric field integral equation (T-EFIE/N-EFIE) and the tangential/normal magnetic field integral equation (T-MFIE/N-MFIE). Among the multiple possibilities of combination of these equations, the following one has proven to be a stable proposal [8]:

$$\sum_{l=1}^{2} a_l \frac{1}{\eta_l}\text{T-EFIE}_l + \sum_{l=1}^{2} b_l \text{N-MFIE}_l \qquad (1) \qquad\qquad -\sum_{l=1}^{2} c_l \text{N-EFIE}_l + \sum_{l=1}^{2} d_l \eta_l \text{T-MFIE}_l \qquad (2)$$

In equations (1) and (2), η_l is the intrinsic impedance in medium R_l (R_1 and R_2 are the exterior and the interior regions of the material, respectively). Different known formulations can be obtained depending on the selection of the complex combination parameters a_l, b_l, c_l and d_l. This comparative study

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involves the formulations known as Poggio-Miller-Chang-Harrington-Wu-Tsai (PMCHWT) [6], combined tangential formulation (CTF), combined normal formulation (CNF) [8] and electric and magnetic current combined field integral equation (JMCFIE) [7]. PMCHWT and CTF formulations combine only tangential equations with $\{a_l = \eta_l, b_l = 0, c_l = 0, d_l = 1/\eta_l\}$ and $\{a_l = 1, b_l = 0, c_l = 0, d_l = 1\}$, respectively. CNF combines only normal equations with $\{a_l = 0, b_l = 1, c_l = 1, d_l = 0\}$ and JMCFIE combines both tangential and normal equations with $\{a_l = 1, b_l = 1, c_l = 1, d_l = 1\}$.

3. Comparative study

A sphere modeled using Rao-Wilton-Glisson (RWG) basis functions [14] with $k_0 r = 3$, where r denotes the radius, $k_0 = 2\pi/\lambda_0$ is the wave number of the surrounding free-space medium and λ_0 is the free-space wavelength, has been employed. The iterative solution of the matrix system using a restarted (30) GMRES [15] has been obtained. An exhaustive analysis of the eigenvalues arrangement, the condition number, the convergence of the iterative scheme and the accuracy of the radar cross section (RCS) prediction has been carried out. Four mediums with a fixed permeability of $\mu_r = -1$ and different permitivity values have been considered in this analysis (see Table 1).

Table 1: 2-norm condition number (c) of the impedance matrix, number of GMRES iterations (i) for residue $< 10^{-6}$ and RMS error (e_{rms}) of the RCS calculation versus Mie's result for a $k_0 r = 3$ sphere with $\mu_r = -1$ and different ϵ_r values. (*Residue after 500 external iterations.)

$\boldsymbol{\epsilon_r = -3}$	c	i	e_{rms}
JMCFIE	$2.0 \cdot 10^6$	7	2.1%
CNF	$3.8 \cdot 10^7$	$(7.5 \cdot 10^{-5})^*$	1.6%
CTF	$1.1 \cdot 10^6$	12	1.9%
PMCHWT	$1.0 \cdot 10^9$	$(1.8 \cdot 10^{-3})^*$	22.3%
$\boldsymbol{\epsilon_r = -1}$	c	i	e_{rms}
JMCFIE	$8.4 \cdot 10^6$	4	0.56%
CNF	$1.0 \cdot 10^7$	5	7.6%
CTF	$3.9 \cdot 10^{12}$	$(4.1 \cdot 10^{-3})^*$	2.3%
PMCHWT	$1.1 \cdot 10^{13}$	$(1.2 \cdot 10^{-3})^*$	20.7%

$\boldsymbol{\epsilon_r = -3 - 0.3i}$	c	i	e_{rms}
JMCFIE	$1.6 \cdot 10^6$	5	1.3%
CNF	$1.7 \cdot 10^7$	32	0.97%
CTF	$1.0 \cdot 10^6$	10	1.8%
PMCHWT	$2.5 \cdot 10^7$	$(1.4 \cdot 10^{-3})^*$	6.0%
$\boldsymbol{\epsilon_r = -1 - 0.3i}$	c	i	e_{rms}
JMCFIE	$2.7 \cdot 10^6$	3	0.98%
CNF	$8.3 \cdot 10^6$	5	6.6%
CTF	$7.1 \cdot 10^6$	15	0.37%
PMCHWT	$2.0 \cdot 10^8$	$(1.0 \cdot 10^{-4})^*$	0.68%

Let us pay special attention to the block of Table 1 which collects the data obtained for the sphere matched to free space, $\epsilon_r = -1$, $\mu_r = -1$. The JMCFIE formulation shows fast convergence, high accuracy in the RCS prediction and the lowest condition number with regard to the rest of combined formulations. Despite the CNF formulation seems to converge in terms of the iterative solver, a limited accuracy has been obtained. It is worth mentioning that the direct solution of the CTF formulation for this test case provides a poor result and a high error rate (around 25%). There is a strong bond between this unreliable behavior of the CTF formulation and the high condition number of its system matrix shown in Table 1. Also the PMCHWT error shoots up absolutely in this particular case when direct solving is applied.

The remarkable issues extracted from this parametric analysis can be summarized as follows:

- The JMCFIE results show that the studied parameters keep similar in all the examples. This fact is indicative of the JMCFIE stability, whereas great variability can be appreciated in the other formulations results.
- Only the results obtained by means of JMCFIE formulation are satisfactory in the matched case. The CNF formulation shows fast iterative convergence but it lacks accuracy (despite iterative or direct solving is applied). The eigenvalues of CTF and PMCHWT formulations are clustered near the origin in this particular case.

- In general terms, the iterative convergence of SIE gets worse as the distance from the origin to the eigenvalues decreases. We have checked that the eigenvalues move away from the origin when adding losses, so the results get better under these conditions.
- The unweighted magnitude of the combination parameters in PMCHWT formulation leads to high condition numbers and disperse eigenvalues distribution, resulting in slow convergence and inaccurate results when solving iteratively. These key points are along the lines of those reported in [10] and [16].

4. Conclusion

The RCS predicted for LHM spheres by four combined integral formulations that traditionally appear in the literature for analyzing dielectric objects has been contrasted with the Mie's series result. Relevant features as the condition number, the eigenvalues distribution and the iterative response have been analyzed. The stability of the JMCFIE parameters obtained in the course of the study is indicative of its suitability to analyze LHM's. In contrast, the CNF, CTF and PMCHWT formulations have shown an unreliable behavior.

References

[1] V. G. Veselago. The electrodynamics of substances with simultaneously negative values of ϵ and μ. *Soviet Physics USPEKI*, vol. 10, no. 4, pp. 509–514, 1968.

[2] R. A. Shelby, D. R. Smith, and S. Schultz. Experimental verification of a negative index of refraction. *Science*, vol. 292, no. 5514, pp. 77–79, 2001.

[3] N. Engheta and Richard W. Ziolkowski. A positive future for double-negative metamaterials. *IEEE Trans. Microwave Theory Tech.*, vol. 53, no. 4, pp. 1535–1556, 2005.

[4] C. D. Moss, T. M. Grzegorczyk, Y. Zhang, and J. A. Kong. Numerical studies of left-handed materials. *Prog. Electromagn. Res.*, vol. 35, pp. 315–322, 2002.

[5] N. V. Kantartzis, D. L. Sounas, C. S. Antonopoulos, and T. D. Tsiboukis. A wideband adi-fdtd algorithm for the design of double negative metamaterial-based waveguides and antenna substrates. *IEEE Trans. Magn.*, vol. 43, no. 4, pp. 1329–1332, 2007.

[6] J. R. Mautz and R. F. Harrington. Electromagnetic scattering from a homogeneous material body of revolution. *Arch. Elektron. Uebertraeg.*, vol. 33, pp. 71–80, 1979.

[7] P. Ylä-Oijala and M. Taskinen. Application of combined field integral equation for electromagnetic scattering by dielectric and composite objects. *IEEE Trans. Antennas Propagat.*, vol. 53, no. 3, pp. 1168–1173, 2005.

[8] P. Ylä-Oijala, M. Taskinen, and S. Järvenpää. Surface integral equation formulations for solving electromagnetic scattering problems with iterative methods. *Radio Sci.*, vol. 40, no. 6 (RS6002), 2005.

[9] Ö. Ergül and L. Gürel. Comparison of integral-equation formulations for the fast and accurate solution of scattering problems involving dielectric objects with the multilevel fast multipole algorithm. *IEEE Trans. Antennas Propagat.*, vol. 57, no. 1, pp. 176–187, 2009.

[10] Y.A. Liu and W.C. Chew. Stability of surface integral equation for left-handed materials. *IET Microw. Antennas Propag.*, vol. 1, no. 1, pp. 84–89, 2007.

[11] J. Rivero, J. M. Taboada, L. Landesa, F. Obelleiro, and I. García-Tuñón. Surface integral equation formulation for the analysis of left-handed metamaterials. *Optics Express*, vol. 18, no. 15, pp. 15876–15886, 2010.

[12] Ö. Ergül and L. Gürel. Efficient solutions of metamaterial problems using a low-frequency multilevel fast multipole algorithm. *Progress In Electromagnetics Research*, vol. 108, pp.81–99, 2010.

[13] R. F. Harrington. *Field Computation by Moment Method*. IEEE Press, NY, 1993.

[14] S. M. Rao, D. R. Wilton, , and A. W. Glisson. Electromagnetic scattering by surfaces of arbitrary shape. *IEEE Trans. Antennas Propagat.*, vol. 30, no. 3, pp. 409–418, 1982.

[15] Y. Saad and M. Schultz. Gmres: A generalized minimal residual algorithm for solving nonsymmetric linear systems,. *SIAMJ. Sci. Statist. Comput.*, vol. 7, no. 15, pp. 856–869, 1986.

[16] P. Ylä-Oijala and M. Taskinen. Improving conditioning of electromagnetic surface integral equations using normalized field quantities. *IEEE Trans. Antennas Propagat.*, vol. 55, no. 1, 2007.