Saturation of the Resonance of the Complementary-SRR

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Abstract

It is well known that the Split Ring Resonator (SRR) reachs a saturation value of the resonant frequency when its size is scaled down. In other works the complementary SRR was proposed as an electric resonator. Here it is demonstrated that a similar saturation phenomenon happens to the complementary SRR.

1. Introduction

At the beginning metamaterials based on the Split Ring Resonator (SRR) [1] were manufactured in the microwaves range. Shortly after, the SRR resonant frequency was pushed up to the THz domain by scaling down the size of the SRR [2]. The next challenge was to achive the infrared and optical regions. However, soon it was clear that the resonant response of the SRR saturates [3] due to the fact that the kinetic inductance, which is usually neglected in macroscopic structures, becomes important in comparison to the magnetic inductance for mesoscopic size (typically ∼100 nm). In a different line of research, Falcone et al. [4] proposed the Complementary-SRR (C-SRR) which is the complementary screen of the SRR. Based on the Babinet's principle, it was pointed out the dual behaviors of these particle. A flat array of SRRs gives a stopband while C-SRRs gives a passband, both very selective and centered at the same frequency. Recently, the Babinet's principle for SRR and C-SRR has been experimentally tested in the near infrared [5, 6]. Although there are losses in the metal, the thickness is finite and some substrate is present, it was found that Babinet's principle is still aproximatelly valid. In this work, the saturation of the resonant response of the C-SRR is studied. Two basic questions rise: Is there a similar saturation phenomenon for the C-SRR? And, if yes, does the C-SRR saturate at a smaller or bigger frequency than SRR does?

2. Theory

For the sake of ease, instead of using the original SRR formed by two metal rings [1], we used the resonators shown in Fig. 1 with only a single ring or a single slot for the C-SRR. It was demonstrated in Ref. [3] that the saturation resonant frequency of the SRR is given by $\omega_s = 1/\sqrt{L_{kin}C}$, where L_{kin} is the kinetic inductance and C is the capacitance. Explicit and very accurate formulas for L_{kin} and C can be found in Ref. [7]. For the case of the C-SRR, we can approximate its capacitance by $C' \approx C'_{pp} + C'_{surf}$, where the first term is a parallel plate approximation for the inner region of the slot and the second term is the surface capacitance for a flat circular disk centered inside a hole (see appendix of Ref. [8]). To calculate the kinetic inductance is much more complicated. In principle we

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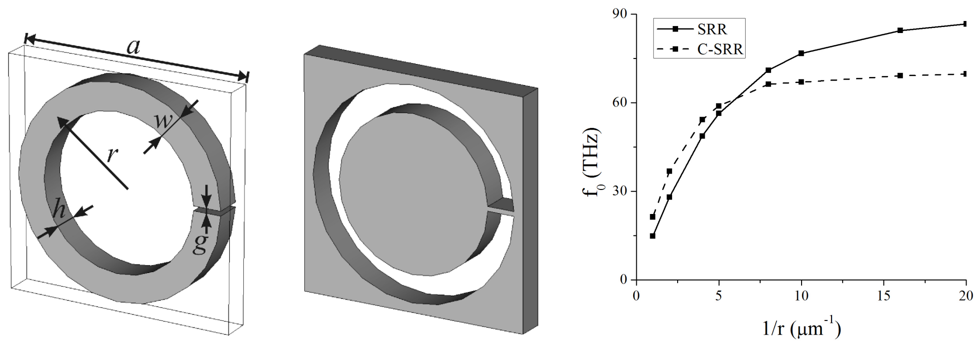

Fig. 1: From left to right, unit cells for metasurfaces made of SRRs and C-SRRs, and plot of resonant frequency versus the inverse radius for both metasurfaces.

need to know how the currents spread in the metal, because a bigger effective transverse section means a smaller kinetic energy. Although we know that the current will be concentrated near the metal edge, its exact distribution is not known until a numerical simulation is done. Our weak hypothesis is that, when the width of the small bridge g is very small, then the bottle neck effect makes the kinetic energy in the small metallic bridge much bigger than the kinetic energy in the open regions of metal (the disk and the hole) which will be thus neglected. Therefore, the kinetic inductance can be approximated by $L'_{kin} \approx L'_{kin,bridge} \approx w/\left(gh\omega_p^2\epsilon_0\right)$.

3. Simulations

In order to validate the theoretical model, a bunch of simulations scaling the size of the SRR and the C-SRR was done. We used the frequency domain solver of the commercial software *CST Microwave Studio*. The used geometrical parameters indicated of Fig. 1 were: $a = 2.5r$, $w = 0.3r$, $h = 0.3r$, and $g = 0.025r$. All are given in terms of the particle radius r which was covering the range between 1 micron and 50 nm. The physical parameters of the metal corresponds with the Drude model for silver: plasma frequency $f_p = 2072$ THz and collision frequency $f_c = 25$ THz. The unit cell was surrounded by periodic conditions. Fig. 1 shows the resonant frequency versus the inverse radius. It is clearly demonstrating a saturation phenomenon for both kind of particles. For the smallest geometry ($r = 50$ nm) the theoretical the model gives us the saturation frequencies $f_s^{SRR} = 99$ THz and $f_s^{C-SRR} = 78$ THz, whose relative separation is in agreement with the values coming from Fig. 1; although there is a frequency shift due to the coupling between neighbours what is not taken into account by the model. From the simulated structure shown here, it could be deduced that the saturation frequency for the C-SRR is smaller than for the SRR. However, it is not neccesarilly so and, in fact, other numerical results for different shapes not shown here demonstrated that the line for C-SRR can also saturate at higher frequency than for SRR. The resonant frequencies were found from the figures of transmission and reflection coefficient (only for normal incidence), being the resonant frequency identified over the transmission spectrum as the dip for SRR and the peak for the C-SRR. For instance, Fig. 2 is showing a subset of transmission and reflection coefficients. It is also apparent that when the C-SRR is approaching the saturation then the response is very attenuated.

4. Conclusions

As a main conclusion, it was found that the resonant response of the C-SRR presents a saturation phenomenon when the size of the particle is continuously scaled down, as similarly happens with the SRR. It

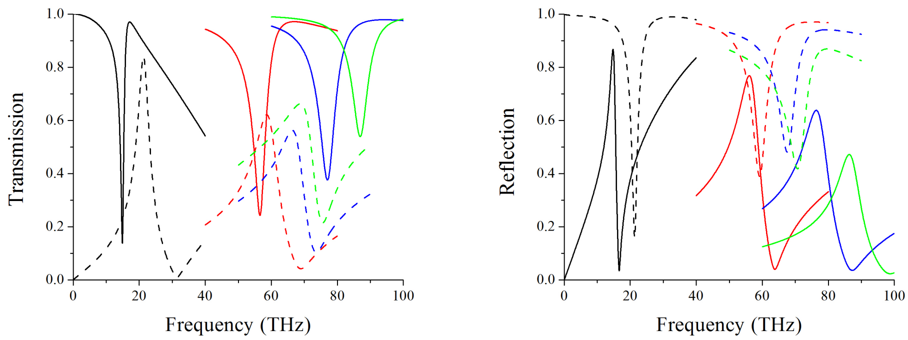

Fig. 2: Scattering parameters for metasurfaces of SRRs (solid lines) and C-SRRs (dashed lines). The inverse radius expressed in μm^{-1} are $r^{-1} = 1$ (black), 5 (red), 10 (blue), and 20 (green).

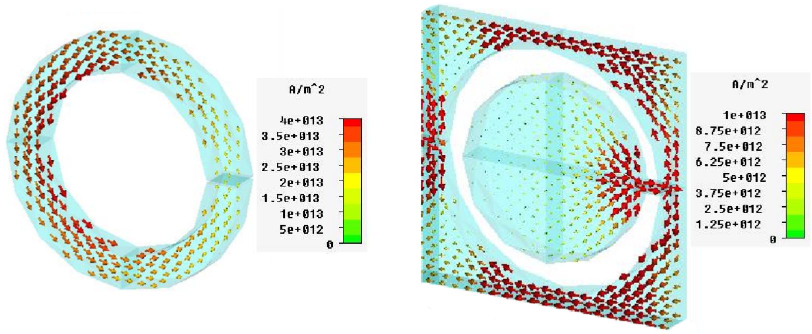

Fig. 3: Electric currents over the SRR and the C-SRR at their resonant frequencies for the case of the smallest resonators with radius $r = 50nm$.

means that the resonant frequency stays below a saturation level and the response in transmission and reflection is strongly attenuated. On the other hand, the saturations of SRR and C-SRR happens at different rates because the Babinet's principle is not valid at all due to the finite thickness and finite conductivity of the metal. We think that this work should be taken into account if a hybrid structure made of SRRs and C-SRRs is desired at optical frequencies.

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