Integrated acoustic and electromagnetic analysis of cylindrical anisotropic metamaterials

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Abstract

Anisotropic characteristics of cylindrically corrugated microstructures are analyzed in terms of their acoustic and electromagnetic behaviours paying special attention to their differences and similarities. A simple analytical model has been developed using effective medium theory to understand the anisotropic features of both types of waves in terms of radial and angular components of the wave propagation velocity. The anisotropic constituent parameters have been obtained by measuring the resonances of cylindrical cavities, as well as from numerical simulations. This permits to characterize propagation of acoustic and electromagnetic waves and to compare the fundamental anisotropic features generated by the corrugated effective medium. Anisotropic coefficients match approximately in both application fields but other relevant parameters explain significant differences in the behaviour of both types of waves.

1. Introduction

Acoustics and electromagnetism are governed by different wave propagation equations that respectively describe sound pressure and electromagnetic field distributions. Key parameters that allow the solution of these problems are on the one hand mass density and bulk modulus and, on the other hand, permittivity and permeability functions. Equivalencies have been already pointed out between both fields that necessarily have to take into account that the sound problem is a scalar one, whereas the light problem is a vector one, [1]. Corrugated surfaces and devices based on corrugations are a direct link between both areas for an integrated and multidisciplinary approach, since they have been analyzed from each point of view for a long time.

2. Experimental characterization

The employed frequency ranges are 1 KHz to 5 KHz for the acoustic waves and 1 GHz to 5 GHz for the microwave experiments. They cover approximately similar wavelength ranges from λ_{min} = 60 mm to λ_{max} = 340 mm. Corrugated structures are basically implemented with a periodically varying stepped surface, usually patterned on metallic materials. An example of a circular metallic cavity with a cylindrically corrugated bottom surface is displayed in Fig. 1, together with a schematic of the measurement setup employed.

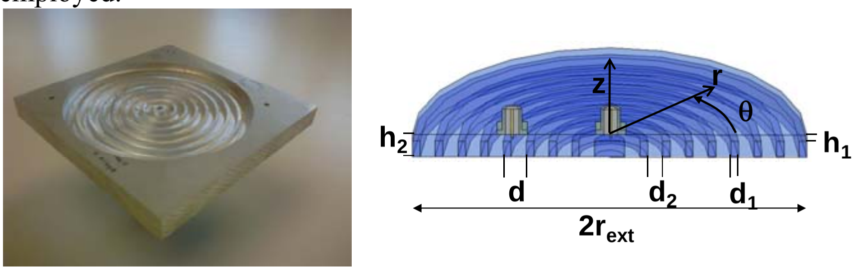

Fig. 1: Left panel: corrugated metallic cavity (open to allow inner vision); Right panel: schematic cross section view of the prototype device with characteristic dimensions and excitation-probing microwave elements.

An initial analysis of the experimental results gives us a certain number of indications, see Fig. 2. Main observable result is the red shift of the resonant frequencies derived from the inclusion of the cylindrical corrugations in the cavity. The frequency displacement is not homogeneous for all modes

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and frequency order of these modes is not equal for both characterizations. In the acoustic case, measurements are performed at a non-central position in the cavity. The monopolar resonance (m = 0) is shifted to lower frequencies as the value of h_1 increases. The other modes (m = 1, 2, 3) are less affected by the introduction of the corrugation. Also, note that relative positions of the different modes are not exactly the same; in particular, first two modes of each regime have swapped positions.

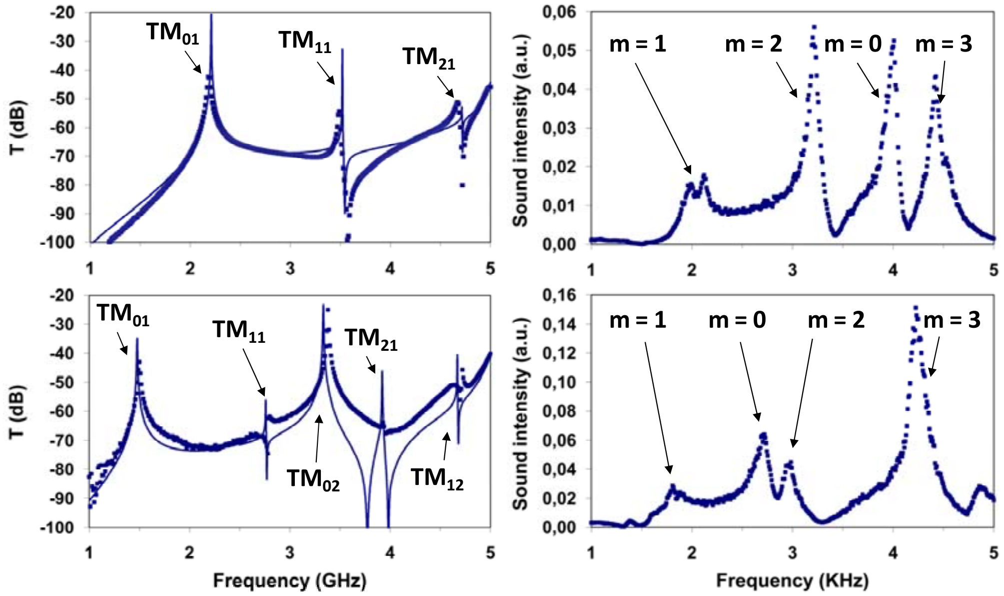

Fig. 2: Resonant frequencies displayed as measured field intensity as a function of frequency. Left panels display the EM case (including numerical simulation results with lines); right panels display measured acoustic results. Upper panels correspond to an empty cavity and lower panels correspond to an h_1 = 2 mm corrugated cavity.

3. Analysis and discussion

Our treatment is based on the characterization of the devices through an effective medium theory. Effective values of radial and angular wave propagation velocities can be extracted from the measured resonant frequencies by analyzing the field expressions of the closed cavities. The equations governing both problems can be expressed in cylindrical coordinates for anisotropic media, [2], respectively for electromagnetic (electric field) and acoustic (sound pressure) waves as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{\mu_\theta}\frac{\partial E_z}{\partial r}\right)+\frac{1}{r^2\mu_r}\frac{\partial^2 E_z}{\partial \theta^2}+\ \omega^2\varepsilon_z E_z=0, \qquad (1)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{\rho_r}\frac{\partial P}{\partial r}\right)+\frac{1}{r^2\rho_\theta}\frac{\partial^2 P}{\partial \theta^2}+\ \omega^2 B^{-1}P=0, \qquad (2)$$

where ω is the angular frequency. The boundary conditions that hold in each case are what basically make the difference between both wave problems. The devices are analysed from the electromagnetic point of view for TM_{mn}^z waves. This polarization configuration is equivalent to the scalar solution of the acoustic problem for the same device. The fact that h_2 is small precludes the possibility of having TE solutions. The cylindrical electromagnetic TM^z problem has solutions given by the Bessel functions *J(x)*, and the resonant frequencies of the different modes are related to the *zeros of the Bessel functions*. Alternatively, the sound problem has solutions given by the derivatives of the Bessel functions *J'(x)*. In this case, resonant frequencies are related to the *zeros of the derivatives of the Bessel functions*. This explains the different relative positions of the lower frequency modes in Fig. 2.
A simple analytical model based on assuming that the height ratio between the empty (noted with sub-index 2) and corrugated (noted with sub-index 1) layers is translated to the effective permittivity values, $\varepsilon_1 h_1 = \varepsilon_2 h_2$. This gives, as derived in [3,4], an anisotropy coefficient calculated between both principal propagation directions (k_θ and k_r), which is equivalent in both problems:

$$\gamma^2 \equiv \rho_r \rho_\theta^{-1} \equiv \frac{\mu_\theta}{\mu_r} = \frac{1}{d^2}\left(d_1 \frac{h_1}{h_2} + d_2\right)\left(d_1 \frac{h_2}{h_1} + d_2\right). \quad (3)$$

Additionally, numerical results based on Comsol simulations are provided, combined with a standard retrieval technique, [6]. Left panel of Fig. 3 displays measured and simulated resonant EM resonant frequencies, with a good agreement. Right panel summarizes the EM parameter extractions performed from three points of view: measurements, numerical results and analytical model. Wave propagation velocity results are normalized with respect to the speed of light in vacuum c_0. The predicted anisotropy from the numerical results is quite close to the parameters extracted from the measured frequencies. Radial effective index, represented again by γ, is calculated as the square root of the product of μ_θ and ε_z. The values of γ follow a decreasing trend as h_1 is increased which is consistent with the predicted behaviour from the analytical model. Both curves do not match exactly basically due to the simplicity of the analytical model that rather gives a qualitative approximation. It is important to note that these numerical and experimental results do match approximately the ones measured in the acoustic case [3]. Calculation of the experimental angular wave velocity is here also determined by $c_\theta = \gamma\, c_r$. The fact that this c_θ result recovers the vacuum light velocity $\tilde{c}_0$ supports the extraction approaches.

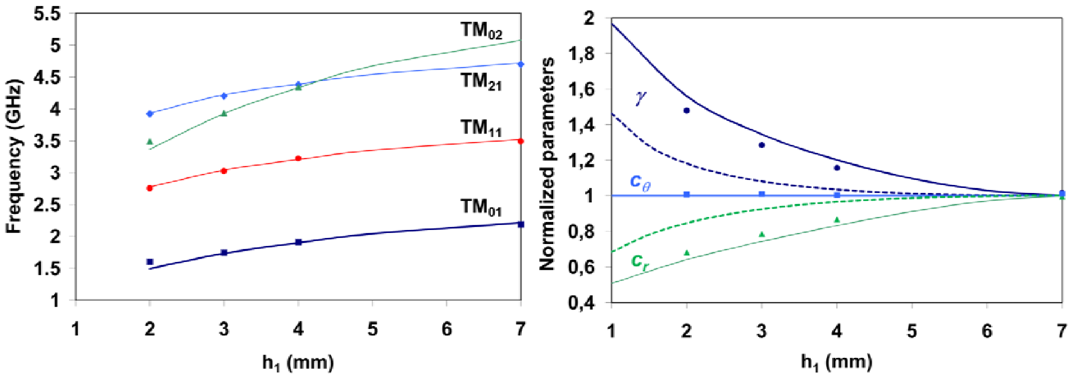

Fig. 3: Left panel: measured (symbols) and simulated (lines) resonant frequencies for the different EM resonant modes in the corrugated cavities. Right panel: Extracted EM effective parameters from the measured results (symbols), numerical simulations (solid lines) and analytical model (dashed lines).

4. Conclusion

A multidisciplinary analysis of resonant cylindrical cavities with anisotropic characteristics has been performed from both acoustic and electromagnetic points of view. The treatment employed is based on the characterization of the devices through an effective medium theory. This procedure allows comparing effective medium parameters in terms of radial and angular wave propagation velocities between both application fields that define an anisotropic behavior. This multidisciplinary analysis may open a path for the design of devices based on an integrated approach to both types of phenomena.

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