Engineering antenna radiation patterns via quasi-conformal transformations

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Abstract

We use a combination of conformal and quasi-conformal mappings to engineer isotropic optical devices that modify the omnidirectional radiation pattern of a point source. For TE waves, the designed devices are also non-magnetic. The flexibility of the proposed technique, higher than that achieved with conformal mappings, is illustrated with some examples.

1. Introduction

Transformation optics makes it possible to design optical media that, when placed in physical space, makes light propagation experience a different space, called virtual space. Points in both spaces (which can be flat or curved) are related by a certain mapping that, together with the metric of those spaces, determines the properties of such optical media [1]. Transformation optics has allowed the design of invisible and directional antennas [2, 3]. Recently, the use of conformal transformations has been proposed to achieve a special class of symmetric two-dimensional directional antennas [4]. Such transformations have the advantage of requiring only isotropic media for their implementation and, for TE polarization, only non-magnetic media [1,4]. Specifically, Schwarz-Christoffel transformations were used to map the circle onto a regular polygon with N sides. This device distributes equally the power of a point source located at the polygon center among N directional beams perpendicular to each side of the polygon, Therefore, this technique is limited to the design of symmetric antennas radiating in N discrete directions. In this work, we combine this kind of conformal transformations with quasi-conformal mappings to gain more flexibility in the design of radiation-pattern-shaping devices.

2. Quasi-conformal mappings for antennas

Conformal mappings transform infinitesimal balls to scaled and rotated infinitesimal balls. This is the reason why they give rise to isotropic transformation media. Quasi-conformal mappings transform infinitesimal balls to ellipsoids of bounded eccentricity. Thus, transformation media resulting from a quasi-conformal mapping have a bounded anisotropy that can be neglected if it small enough. Our goal is to change the omnidirectional radiation pattern of a two-dimensional point source. For this purpose, we will consider the transformation of the unit circle in a flat virtual space to another shape in flat physical space. We can achieve this transformation in two steps. First, we transform the unit circle to the unit square by using a combination of a Möbius transformation mapping the circle to the half upper plane, followed by a Schwarz-Christoffel transformation mapping the half upper plane to the unit square. The complete transformation is given by:

$$q(w) = \sqrt{-2i}\left(1 - \frac{1}{F\left(\pi/2\,|\,1/2\right)} F\left(\frac{\pi}{2} - \arcsin\left(w\sqrt{i}\right)\middle|\frac{1}{2}\right)\right) \quad (1)$$

Where $F(\varphi|m)$ is the incomplete elliptic integral of the first kind, with amplitude φ and parameter m. We have expressed this two-dimensional transformation as a function of the complex variable $w = w_1 + iw_2$, with $q = q_1 + iq_2$. The refractive index that implements this transformation can be found in [4]. As for the second step, we use a quasi-conformal mapping to transform the unit square to the desired final shape. The advantage of quasi-conformal transformations of the unit square is that, unlike conformal ones, they can be easily calculated numerically and they always exist. We only need to be careful so that this mapping has an associated negligible anisotropy and can be approximated by a conformal mapping $z(q)$. Here we will use a simple way of computing such quasi-conformal mappings, which is based on the solution of an inverse Laplace equation supplemented with sliding boundary conditions [5]. In this case, the four sides of the square are mapped to four disjoint specified pieces of the transformed square boundary. The complete transformation refractive index is given by $n = |dw/dq|\cdot|dq/dz|$ [1]. In Fig. 1 we illustrate the two steps of this transformation with an example.

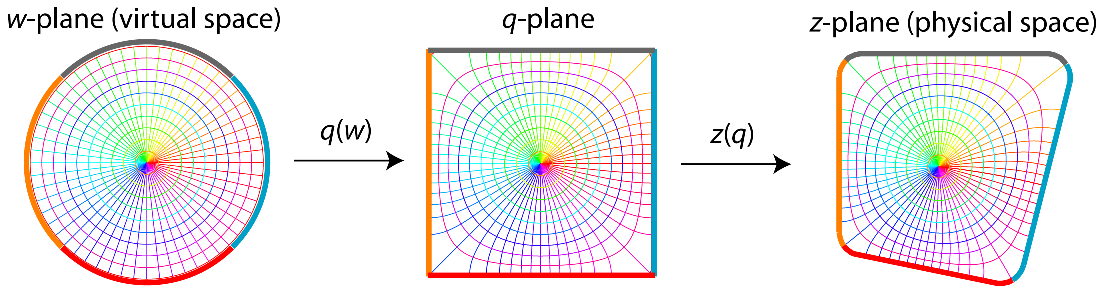

Fig. 1. Mapping of the unit circle to another shape in two steps.

Conformal maps preserve angles, *i.e.*, two curves meeting at a certain angle in virtual space are mapped to curves in physical space that meet at the same angle. Lines perpendicular to the unit circle boundary will be perpendicular to its transformed counterpart in physical space. Electromagnetic fields follow this transformation, so light rays emanating from a point source located at the center of the unit circle in original space will be normal to the transformed boundary in physical space as well. In order to shape the radiation diagram of the omnidirectional source, we have to orient each little piece of the transformed boundary so that it is perpendicular to the direction we want to redirect the rays crossing that piece. This way we can engineer the angular distribution of the radiated power. This procedure is not exact because of the wave nature of light and the reflections appearing at the transformed circle boundary, since the transformation is not continuous at it. The other limitation is that we do not have full control of the density of rays crossing the transformed circle boundary. We can only decide where to map each fourth of the circle boundary so that we can distribute the radiated power among four desired sets of angular directions, but we cannot specify the angular distribution within each set. Despite the first limitation, the results achieved by this technique are quite accurate. In addition, the second limitation can be overcome to a certain extent as shown below, increasing the degree of control of the angular power distribution. In Fig. 2 we depict three examples illustrating the potential of this technique. We focus on TE waves so that our devices can be implemented with an index gradient. In the first column we show how each mapping transforms the grid in the w-plane depicted in Fig. 1. In the second column we include the refractive index that implements such mapping. In the third column, we render the power distribution (calculated with COMSOL Multiphysics) of a point source located in the transformed center of the circle. Finally in the fourth column we depict the directivity of the resulting antenna. As a first example, imagine that we want to divide the point source radiated power into four directional beams, each one propagating in an arbitrary direction. To this end, we should use a quasi-conformal mapping transforming each side of the square to a straight line perpendicular to each of these directions, as in Fig. 2(a). The directivity diagram confirms the desired result. Note that not all lobes have the same directivity due to the above-mentioned limitations. This could be improved by optimizing the mapping. In the second example we show that this technique is not restricted to four-beam antennas. In this case we assign the left side of the square to two segments, each perpendicular to a different direction. Since they are symmetric, we know that the beams exiting

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each of them will carry the same power, approximately a quarter of the power of the other three beams. In the third example, we show that we can also engineer the device to have isotropic radiation for a certain angular range, and not only a set of directional beams. In this case, the upper side of the unit square is transformed to a circular boundary, giving rise to an approximate isotropic radiation in a 60º region with directivity around 1.5. Finally, note that the quasi-conformal mapping flexibility enables us to avoid steep vertices so that the required refractive index is always greater than zero.

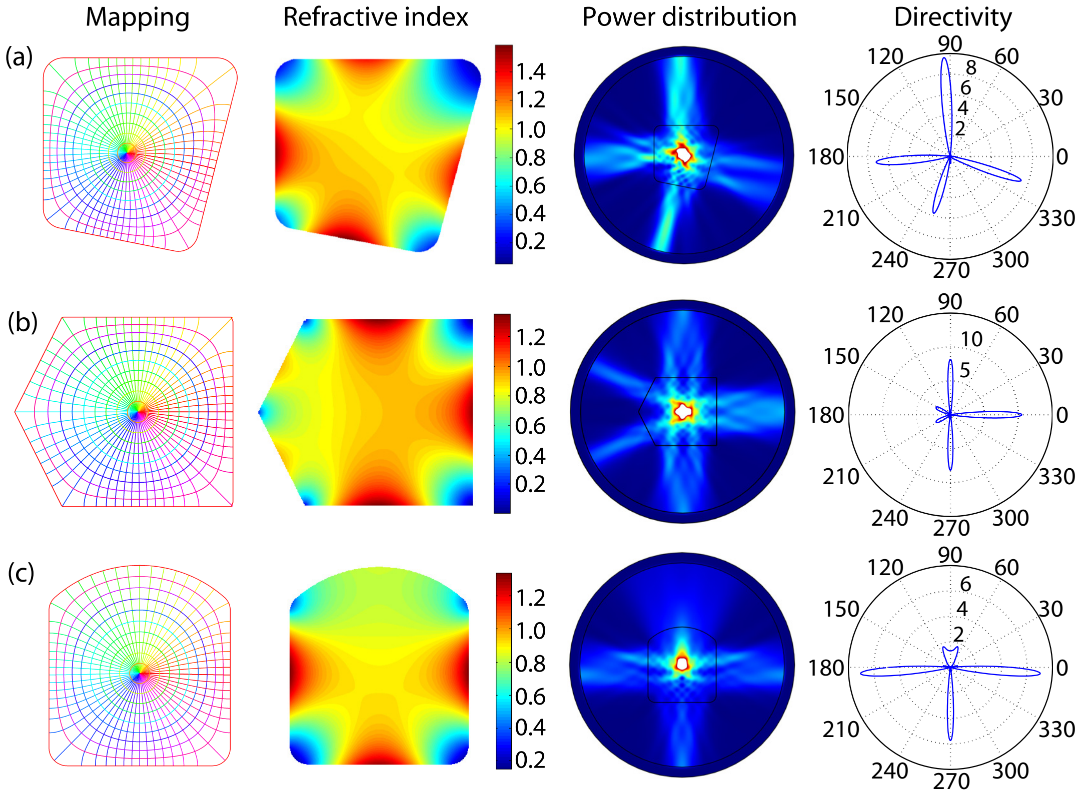

Fig. 2. Three different examples of radiation-pattern-shaping devices based on quasi-conformal mappings.

3. Conclusion

We have shown how to engineer antenna radiation patterns in several ways with the aid of quasi-conformal mappings. This technique provides us with a good degree of control, allowing us to divide the power into highly directional beams in a set of desired directions and isotropic radiation in other angular ranges. More complex radiation patterns could be achieved by combining the presented ideas. The proposed devices are isotropic and non-magnetic. The flexibility offered by quasi-conformal mappings enables us to avoid zero-index regions. Financial support by the Spanish MICINN (contract CSD2008-00066 and FPU grant) is gratefully acknowledged.

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