All Kinds of Cloaks, all Kinds of Transformations: a general theory of *Transformation Mechanics*

Paul Kinsler, Martin W. McCall, Alberto Favaro

Blackett Laboratory, Imperial College London,
Prince Consort Road, London SW7 2AZ, United Kingdom.
Fax: + 44 20 759 47714; email: p.kinsler@imperial.ac.uk

Abstract

The diverse collection of electromagnetic cloaks includes object, carpet, and exterior cloaks–even space-time “event” cloaks. Other transformation optics devices are field-concentrators, super-scatterers, and ray-geodesic imagers. We show how to generalize transformation optics, and make it applicable to *any* kind of wave propagation, such as acoustic or water waves.

1. Spatial Transformation Optics

Electromagnetic cloaking was the first example of a “transformation optics” that manipulated the spatial properties of a propagation medium in a way calculated to mimic a coordinate transformation. Thus it used the invariance properties of Maxwell’s equations to determine how coordinate deformations could be actualized in an electromagnetic medium [1]. The concept was experimentally realized shortly thereafter [2], where a 2D spatial cloak was created that was capable of hiding an interior region of space from observers. Ongoing progress towards an ideal macroscopic wide-bandwidth electromagnetic cloak continues to be a major goal for the experimentalists. Variants include carpet cloaks which hide objects in reflection, e.g. the 3D metamaterial implementation of [3], and (rather more simply) the use of polarization selectivity using calcite [4] to make a 1D carpet cloak in the visible spectrum. Another cloak variant is that of exterior cloaks, where a cloaking device is designed so as to cancel the scattering from some spatially separated object.

2. Space-time Transformation Optics

Existing transformation optics technique were recently extended into the space-time domain, with the proposal for a space-time event cloak [5], that, at least conceptually, has the remarkable capability of acting as a history editor. This is a significant extension of the purely spatial concept of Pendry, since now the coordinate transformations embrace time as well as space. Thus the electromagnetic medium for realizing the desired space-time transformations must be dynamic as well as inhomogeneous. By influencing the local speed of light, the event cloak ensures that any light reaching a surveillance camera can record only an edited version of the events in the area it watches.

One new requirement of the event cloak is that it regulate the local speed of light; thus the average value must be less than the vacuum light speed. In transformation optics terms, this means that the coordinate transformation is carried out in a medium with uniform refractive index, not vacuum.

Thus space-time transformation optics is already a non trivial extension of the original programme which has hitherto only applied spatial coordinate transformations to vacuum. Further, using quite general

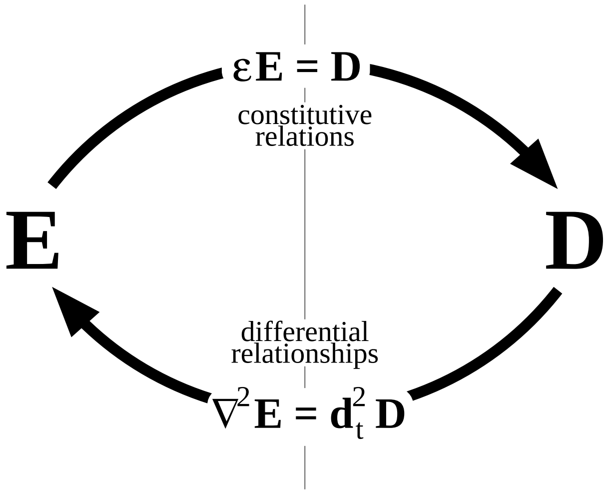

Fig. 1: Schematic representation of a Transformation Mechanics theory with fields E and D related by (a) a constitutive relation defining the host medium (here defined by e.g. just ε, with $\mu = 1$), and (b) an equation defining the differential relationships between the coordinate axes (here e.g. the wave equation $\nabla^2 E = d_t^2 D$). Although the symbols and example relationships have been chosen to match those in a simple electromagnetic transformation mechanics, many variants are possible. If the modification in the differential relationships due to a coordinate change is counter-balanced by a complementary change in the constitutive relation, we have achieved our goal.

methods, we have shown how such a generalization can be embedded within a general theory of non-birefringent transformations [6].

3. Transformation Mechanics

Beyond electromagnetic situations, transformation theory has also been applied now to acoustics to produce a variety of interesting and experimentally realizable schemes [7, 8]. Although acoustics is *not* mathematically equivalent to electromagnetics [9], the ideas of actualizing a coordinate transformation do carry over to acoustic wave propagation. Indeed, any appropriate system of equations – e.g. those for water waves – allow leeway for a transformation mechanics of the kind already applied in optics and acoustics.

Further, what if the initial medium is nonlinear or otherwise programmable? Influencing the initial medium will influence the functionality of the final medium in ways that have yet to be explored. Here controllable cloaking is an obvious goal, but a more general formalism will lead to the potential for more complex and subtle optical machines. As an example, we consider whether a nonlinear transformation media can map directly onto the weak-gravity limit of general relativity.

We have therefore asked ourselves several questions: What are the essential ingredients of any transformation theory, and to what other areas of physics can such schemes be applied? What are the distinctions between and relative capabilities of transformation theories that apply to waves in scalar, vector and tensor fields? In answer to these questions we will describe the structure and relationship of propagation and constitutive equations, and derive a general theory of *transformation mechanics*. This can then be specialized to the field appropriate to one's interests – whether that be acoustics, electromagnetism and optics, water waves, or something else.

4. Summary

We will categorize each existing type of transformation optics or transformation acoustics device, whether based on purely spatial or space-time coordinate transformations. This then leads us to the potential of dynamic and nonlinear transformation devices, and finally allows us to realise a very general theory of *transformation mechanics*.

Acknowledgments: The authors acknowledge financial support from both a Leverhulme Trust *Embedding of Emerging Disciplines* award (2009) and the EPSRC (EP/E031463/1).

References

[1] Pendry, J.B., Schurig, D. & Smith, D. R., Controlling Electromagnetic Fields, *Science* **312**, 1780-1782 (2006).

[2] Schurig, D., Mock, J. J., Justice, B. J., Cummer, S. A., Pendry, J. B., Starr, A. F. & Smith, D. R. Metamaterial Electromagnetic Cloak at Microwave Frequencies, *Science* **314**, 977-980 (2006).

[3] Ergin, T., Stenger, N., Brenner, P., Pendry, J.B. & Wegener, M., Three Dimensional Invisibility Cloaking at Optical Wavelengths, *Science* **328**, 337-339 (2010).

[4] Chen, X., Luo, Y., Zhang, J., Jiang, K., Pendry, J.B. & Zhang, S., Macroscopic Invisibility Cloaking of Visible Light, *Nature Communications* **2**, 176, (2011).

[5] McCall, M.W., Favaro, A., Kinsler, P. & Boardman, A.D., A Spacetime Cloak, or a History Editor, *Journal of Optics*, **13**, 024003, (2011).

[6] Favaro, A. & Bergamin, L., The Non-birefringent Limit of all Linear, Skewonless Media and its Unique Light-cone Structure, *Ann. Phys. (Berlin)*, 119, (2011).

[7] Zhang, S., Xia, C. & Fang, N., Broadband Acoustic Cloak for Ultrasound Waves, *Phys. Rev. Lett.*, 024301, (2011).

[8] Torrent, D. and Sanchez-Dehesa, J., Acoustic Metamaterials for New Two-Dimensional Sonic Devices, *New J. Phys.*, **9**, 323, (2007).

[9] Cummer, S. A. & Schurig, D., One Path to Acoustic Cloaking, *New J. Phys.*, **9**, 45, (2007).