Conical refraction for new classes of biaxial metamaterials

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Abstract

The study of conical refraction for media other than biaxial non-magnetic crystals has never been considered in the literature – as far as the authors are aware. Hence, in this paper, we investigate how conical refraction can arise for broader classes of biaxial metamaterials using a new classification scheme previously developed by the authors. Namely, conical refraction is shown to occur for media with both electric and magnetic anisotropies. Furthermore, this same effect is analyzed for a special class of biaxial bianisotropic media.

1. Introduction

The phenomenon of conical refraction is an ancient topic of optics, theoretically discovered in 1832 by Hamilton. In the same year, Lloyd experimentally demonstrated this effect using a rare specimen of arragonite. This was a remarkable experiment which was very difficult to reproduce due to the lack of suitable crystals. Only in 2004, the engineers of the company Vision Crystal Technology AG found an easy setup to demonstrate conical refraction using a monoclinic double tungstate crystal. With the flexibility brought by artificial crystals, a new interest was regained in this old topic of optics, and new applications for conical refraction are now being under investigation [1],[2]. On the other hand, with the advent of metamaterials more general classes of anisotropic media are now being studied. Namely, media with simultaneous electric and magnetic responses at the same frequency range (either in the microwave or in the optical domains) [3],[4]. The authors have presented a novel approach to address these media based on (Clifford) geometric algebra [5],[6]. Using this new coordinate-free approach, we defined a broader class of biaxial media. However, in this analysis an important question was left unsolved: does conical refraction occurs for these biaxial media? Thus, the first milestone of this communication is to show that a more general type of conical refraction can occur for biaxial materials with both electric and magnetic anisotropies. We obtain the general expression for the light cone that depends on the electric and magnetic properties of the material, and not only on the electric parameters, as is for the conventional biaxial case. Furthermore, we should stress that a biaxial behaviour is not restricted to anisotropic media. In fact, we introduce a new class of bianisotropic media that have two optic axes, i.e. two directions for the wave vector in which the refractive indices of the two eigenwaves are the same. One should not confuse the physical definition of biaxiality with the mathematical biaxial form of the constitutive tensors. As we have already shown, global biaxiality can be obtained from mathematical uniaxial constitutive tensors. Clifford geometric algebra plays the central role of providing a new geometric insight capable of predicting this new class of bianisotropic media based on the analysis developed for anisotropic media. Using this framework, we will address if conical refraction, or other more general kind of refraction, can manifest in these biaxial bianisotropic media. This work intends to show that conical refraction is not a simple theoretical curiosity restricted to conventional biaxial crystal. In fact, we believe that this physical effect can bring new possibilities to this ongoing area of research that is metamaterials.

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2. Conical Refraction

The analysis of monochromatic plane wave propagation in an unbounded source-free region leads to a homogenous characteristic equation in terms of the electric or magnetic fields. This equation follows from the Maxwell Equations and the constitutive relations. The solution of this equation characterizes the eigenwaves of the medium in terms of the refractive index and polarization. Nevertheless, in some special cases, as it occurs when the wave vector has the optic axes direction, this equation does define a unique polarization for each eigenwave. This implies that the Poynting vector instead of having a defined direction, describes a surface that in the case of a biaxial anisotropic crystal is a cone. In Fig. 1, we show a more general light cone considering biaxial media with both electric and magnetic anisotropies. We should stress that the example given in Fig. 1 represents a general biaxial medium composed by a biaxial permittivity and a uniaxial permeability.

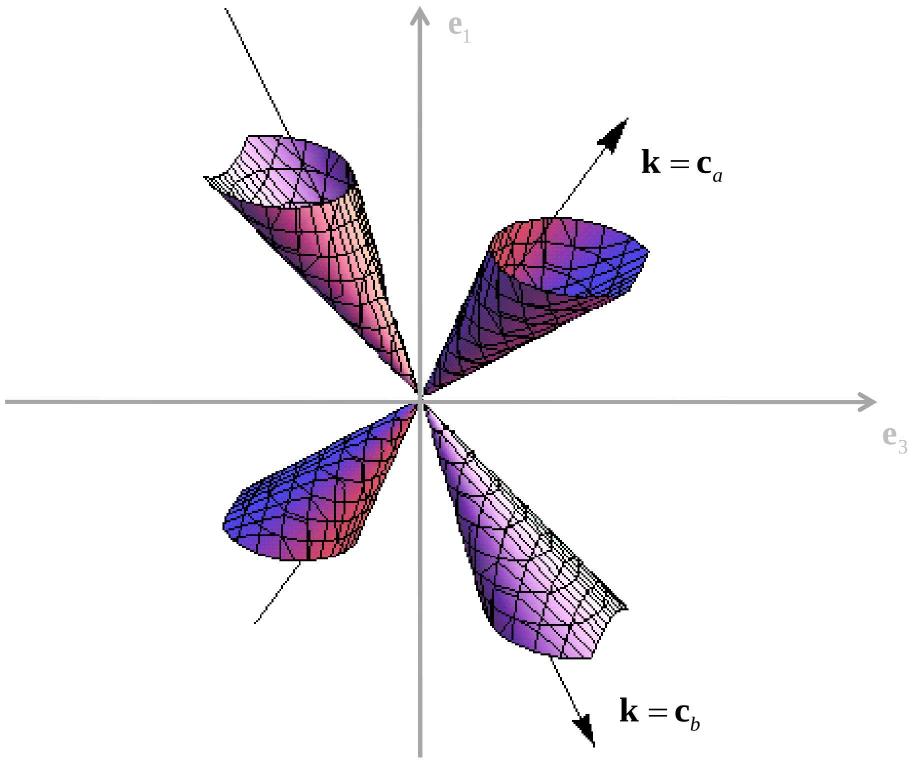

Fig. 1: Representation of the cone generated by the ray vector of the eigenwave for a anisotropic medium characterized by the constitutive parameters: $\varepsilon_1 = 2\ \varepsilon_2 = 3/2\ \varepsilon_3 = 1$ and $\mu_1 = \mu_2 = 2,\ \mu_3 = 3$.

In this communication, we will present an analogous analysis but considering a new class of biaxial media that is bianisotropic. In fact, we found that a pseudochiral omega medium [8], under certain restrictions, can have global biaxial behavior (Fig. 2). We should note that in general these media do not have optic axes. It was through a new geometric approach based on Clifford algebra that was possible to derive straightforward geometric relations that guarantees the existence of optic axes.

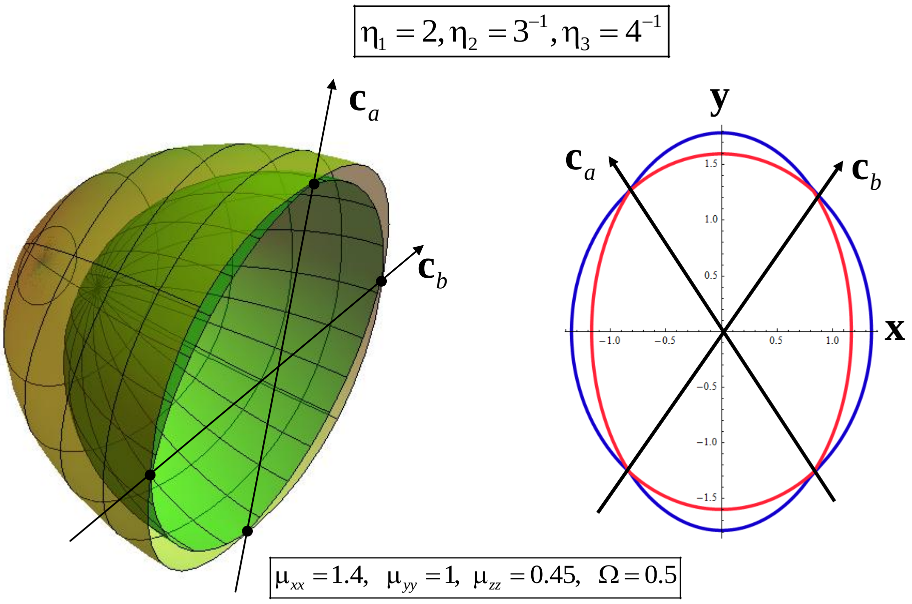

Fig. 2: Refractive index surfaces of a biaxial pseudochiral omega medium where $\eta_i = \mu_i / \varepsilon_i$ with $i = 1,2,3$.

3. Conclusions

In this communication we have analyzed how conical refraction occurs for two general biaxial media: i) a biaxial anisotropic medium characterized by having both electric and magnetic anisotropies; ii) a biaxial pseudochiral omega medium that was introduced by the authors in this work. Hence, it was shown that conical refraction is not an effect restricted to electric anisotropy. In fact, this physical effect can be found when magnetic and electric anisotropic simultaneous take place or even for bianisotropic media.

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