Resonant Frequencies of Split Ring Resonator in Respect of Angle between Slits

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Abstract

In this paper we proposed the method for calculation of resonant frequencies of split-ring resonators (SRRs) as a function of the angle between slits. Method is based on transmission line (TL) approach and can be used for calculation of resonant frequencies of broadside and edge-coupled, as well as of multiple SRRs. Resonant frequencies are also calculated using 3D EM simulations based on method of moments and compared with our model. It is shown that very good agreement is achieved in case when substrate thickness is smaller. Limitations of the model are discussed.

1. Introduction

Split ring resonator (SRR) is probably the most popular element used in metamaterials. It was introduced by Pendry [1], who showed that it can produce negative effective permeability near its resonant frequency–the magnetic plasma effect. Therefore knowing these resonances is of vital importance for metamaterial design. Generally, a good analytical model of SRR would provide a valuable insight in properties and behaviour of metamaterials, and help to facilitate optimization process. A summary of analytical formulae for resonant frequencies for various SRR configurations, compared with numerical calculations and measurement is given in [2].

Our interest in SRR, and, particularly, shifting of resonances with respect to angle between slits is derived from our previous work [3], where we investigated left-handed microstrip transmission line loaded with SRRs, and showed that frequency bands of ‘left-handedness’ shift when we rotate particular rings. As this rotation is nothing else than changing the slit position, it could also be realized via electrically controlled PIN diodes, thus offering the potential for electronic tunability.

2. Transmission line model of SRR

We shall note firstly that by SRR we consider a system of two metallic rings, placed either concentrically (edge-coupled) or one above the other (broadside-coupled). Each ring has slits at certain angle–this is sometimes referred as doubly-split double ring or DSDR. Frequently used model for one ring is resonant LC circuit, and for double ring–two coupled LC circuits. However, we find this model is not good enough when circumference of the ring becomes comparable to the wavelength, and experimental and numerical results show this happens already at the first resonance.

A model based on distributed circuits was proposed by Shamonin et al. in [4], and it reportedly shows good agreement not only for first, but also for higher resonances. In their approach, section of SRR is treated similarly as section of two-conductor transmission line. Namely, they use circuit model for one small section, corresponding to angle $d\varphi$, which includes mutual and self inductances and only mutual capacitance.

Here we propose transmission line model which includes both mutual and self inductances and capacitances. This is necessary for the SRR over ground plane beneath – the requirement of microstrip tech-

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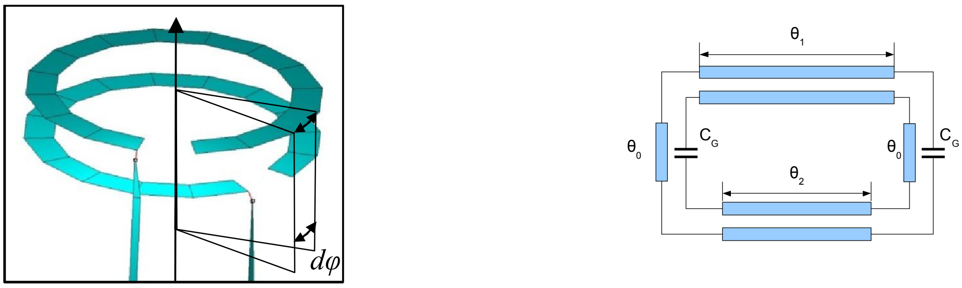
nology. Because of that, SRR section includes third conductor – ground – and becomes identical to that of a two-conductor line. Therefore, we conclude that we can model SRR with two stubs of two-conductor line with appropriate parameters and electrical lengths θ_1 and θ_2, connected in loop with capacitors, corresponding to gap capacitances C_g and single-conductor line with electrical length θ_0, as depicted on Fig. 1b.



(a) (b)

Fig. 1: (a) Model of broadside-coupled SRR for MoM analysis (ground plane omitted for sake of simplicity), (b) Equivalent circuit of SRR using transmission lines.

Wave propagation on transmission line is governed by set of primary line parameters, which in no-loss case are [L] and [C] matrices. We used quasi-static based program LINPAR [5] to obtain these values, setting parameters to match cross-section of SRR – conductor widths, spacing, permittivity of dielectrics etc. Note that only this step is different depending of the type of SRR (broadside or edge-coupled), and once the parameters are obtained, we proceed the same way in both cases.

We inserted [L] and [C] values into the model in Microwave Office (MWO) software. We would like to stress out that this is, in essence, an analytical model, as it is based on closed-form expressions, and, in case of need, calculations could also be performed by hand. To determine resonances, we added ports to each gap and observed transmission between them, i.e. S_{21} parameter.

The fundamental limitation of the proposed model is that it considers coupling only between adjacent sections of SRR, which stems from assumptions for quasi-static approach to transmission lines. In other words, it can't take account of coupling of sections which are, for example, at opposed sides of SRR. We believe that, if spacing between conductors in one section is sufficiently small (compared to the radius of SRR), most of the field would indeed be confined in such section, and thus the non-adjacent coupling can be neglected.

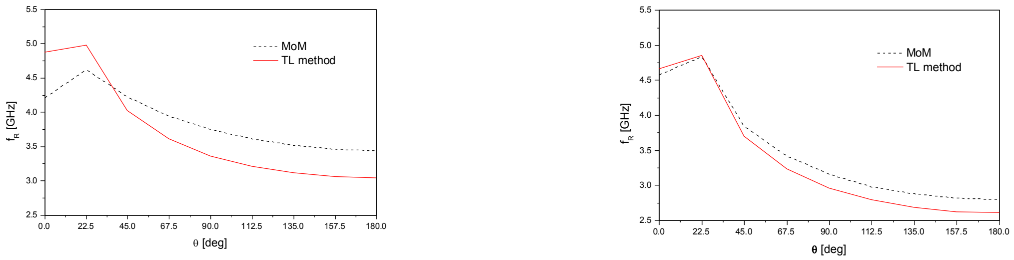
3. Results

We compared results for resonances in MWO with a 3D EM simulation carried out in WIPL-D, based on method of moments. Here we list the results for the first resonant frequency plotted versus the angle between slits, in 22.5° steps.

We started with broadside-coupled SRRs with diameter d=3.35mm, on two-layer substrate with thickness and permittivity ε_1=2.2, h_1=1.5748mm and ε_2=10.2, h_2=0.635mm ($h=h_1+h_2$=2.21mm). Note that these values do not satisfy small height-to-radius condition, but they correspond to parameters used in our earlier work. Results are plotted on Fig. 2a. While there is considerable disagreement, shapes of curves are relatively similar, so we proceeded with taking smaller substrate thicknesses, namely h_1=0.762mm and h_2=0.254mm (h=1.01mm), plotted on Fig. 2b. We can see that agreement has much improved, thus supporting our discussion on limitations of the model.

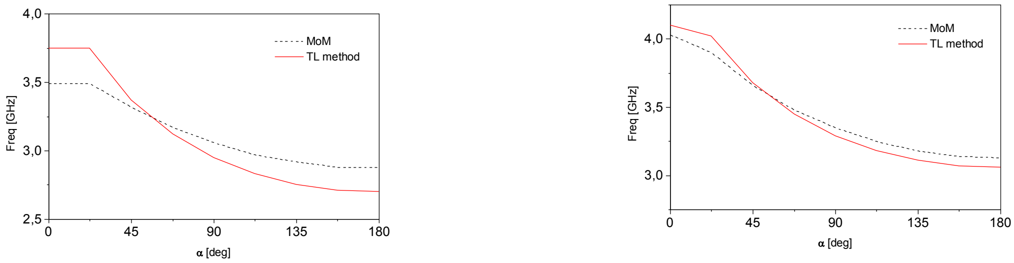
We also tested our model for edge-coupled SRR, using the same two substrates as before, with conductor widths $w_1=w_2$=0.3mm and spacing s=0.1mm. Results are plotted on Fig 3. For this case, matching also improves on thinner substrate, and in general is even better than in broadside-coupled case.

A word on range of frequency shifts, which is one of the main aims of our work. For broadside-coupled SRR on thicker substrate (Fig. 2a), frequency spans from 3.44 to 4.62 GHz, which is relative range of 29% (with respect to central frequency). On thinner substrate (Fig. 2b), it spans from 2.8 to 4.84 GHz or 53%. For edge-coupled SRR on thicker substrate (Fig. 3a) it is 2.88-3.49 GHz or 19%, and for thinner substrate (Fig. 3b) it is 3.13-4.03 GHz or 25%.



(a) (b)

Fig. 2: Broadside-coupled SRRs: (a) thicker substrate (h=2.21mm) and (b) thinner substrate (h=1.01mm).



(a) (b)

Fig. 3: Edge-coupled SRRs: (a) thicker substrate (h=2.21mm) and (b) thinner substrate (h=1.01mm).

4. Conclusion

We demonstrated equivalent-circuit model of SRR based on transmission lines, and applied it to determine resonances in respect of angle between slits. Results are compared with 3D EM simulation. The best agreement is for the case of edge-coupled SRR on thinner substrate, within 3%. Relative range of frequency shifts achieved is 53% for broadside- and 25% for edge-coupled. This difference we attribute to the weaker coupling in the latter.

References

[1] J.B. Pendry, A.J. Holden, D.J. Robins and W.J. Stuart, Magnetism from Conductors and Enhanced Nonlinear Phenomena, *IEEE Trans. Microwave Theory Tech,* vol. 47, p. 2075, 1999.

[2] A. Radkovskaya, M. Shamonin, C.J. Stevens, G. Faulkner, D.J. Edwards, E. Shamonina and L. Solymar, Resonant Frequencies of a Combination of Split Rings: Experimental, Analytical and Numerical Study, *Microwave and Opt. Tech. Lett*, vol. 46, pp. 473-476, 2005.

[3] V. Milosevic, B. Jokanovic and B. Kolundzija, Microwave stereometamaterials and parameter extraction, Proceedings of *Metamaterials'2010*, pp. 474-477, Karlsruhe, Germany, 13-16 September 2010.

[4] M. Shamonin, E. Shamonina, V. Kalinin and L. Solymar, Resonant Frequencies of a Split-Ring Resonator: Analytical Solutions and Numerical Simulations, *Microwave and Opt. Tech. Lett*, vol. 44, pp. 133–136, 2005.

[5] A.R. Djordjevic, M.B. Bazdar, T.K. Sarkar and R.F. Harrington, *LINPAR for Windows: Matrix Parameters for Multiconductor Transmission Lines, Software and User's Manual*, Norwood, USA: Artech House, 1995.