# An Integral Equation Approach to Model Gain Media at Optical Frequencies

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## Abstract

An integral equation approach is being developed to model gain media at optical frequencies. The approach is based on modeling the effect of active optical materials such as organic dye molecules by equivalent sources inside a passive medium under external illumination. As an initial step, the equivalent sources are approximated as electric current sources that are proportional to the tested electric field intensity inside the passive medium due to external illumination at the pumping frequency. A more elaborate model needs to be developed to relate the equivalent sources at the frequency of operation to the tested field at the pumping frequency to represent the effect of dye molecules inside the passive medium.

## 1. Introduction

Optical metamaterials typically exploit the properties of a plasmonic material, such as silver or gold, which exhibits negative permittivity at optical frequencies. However, most of the existing and potential realizations suffer from damping caused by metal absorption. It has been predicted that optical gain can compensate for absorption loss in surface plasmons [1-3]. The enhancement of localized surface plasmons by incorporating an active material, such as organic dye molecules, has been suggested as an effective method to reduce losses in optical metamaterials [3-6].

Integral equation approaches such as the Method of Moments (MoM) are an appropriate technique applied in the numerical analysis of metamaterials because (i) important underlying analytical results are represented in the form of Green's functions, (ii) the radiation condition is fulfilled implicitly and (iii) unknowns are limited to interfaces between homogeneous media. These features allow for fast solution and easy physical interpretation of the results. The remainder of the paper is organized as follows. In Section 2, we describe a formulation for the periodic Green's function and its derivatives, both of which converge exponentially [7]. In Section 3, we describe an integral-equation approach based on the surface equivalence principle to model gain media at optical frequencies. This is shown for doubly periodic infinite arrays of gold nanorods excited by a periodic array of electric point sources. Conclusions are provided in Section 4.

## 2. Periodic Green's Functions

It is well known that periodic Green's functions can be efficiently determined with the help of plane-wave decompositions, leading to exponential convergence, with an exponent proportional to the height of the observation points above the array of dipoles. The on-plane convergence problem can be solved with the help of cylindrical wave decompositions, where cylindrical waves radiated by infinite lines of sources are added together in the space domain, with the help of the Levin extrapolation procedure. The exponential convergence of the space-domain series can be maintained in the low-frequency limit by adaptively augmenting the number of points and the different values of the sum exploited in the extrapolation procedure [7]. The method appears to be even more efficient for the gradient of the Green's functions, which is needed for the analysis of dielectric structures, where the surface equivalence principle is exploited [8].

## 3.Modeling Gain Media

Gain media may surround the lossy plasmonic structures. In this paper, to start with, the dye-based gain molecules will be assumed to be located directly inside the plasmonic particles. Using the surface integral equation approach given in [8], one can compute the total electric field intensity at any point inside the unit cell of doubly periodic infinite arrays of gold (Au) nanorods, illustrated in Fig. 1, excited by a periodic array of electric point sources. This allows the representation of the effect of dye molecules dispersed outside the nanorod by equivalent electric current sources related to the total electric field intensity inside each nanorod. These equivalent sources radiate inside the nanorod; therefore, in order to find the radiated electric field intensity outside the nanorod, one employs the surface equivalence principle to find the induced electric and magnetic current densities on the surface of the nanorod due to the sources inside.

In our numerical study, we can run two simulations: pumping and radiation by equivalent sources that represent dye. As a starting point, we make a crude approximation of the pumping process: we test the electric field intensity at a few points inside the passive medium under the external illumination. The equivalent sources that represent the effect of dye inside the medium are approximated as electric current sources proportional to the electric field intensity observed at the testing points. One can compute the electric field intensity radiated by these sources at any point inside the unit cell. The mathematical formulation of this double process with the help of the Method of Moments will be described in a later communication.

In Fig. 1, the magnitudes of the x and z components of the electric field due to external illumination are plotted on a test plane. The equivalent sources are due to the tested field at ten equally distanced points along the axis of the nanorod. The magnitudes of x and z components of the radiated electric field due to equivalent sources are also shown. For the given aspect ratio of the nanocylinder, the longitudinal localized surface plasmon resonance occurs at $\lambda_o$=552 nm where $\lambda_o$ is the free space wavelength at the frequency of operation. For $\lambda_o$=540 nm and $\lambda_o$=520 nm, the magnitude of the observed electric field intensity decreases gradually. The decrease in the magnitude of the radiated electric field intensity is proportional to the decrease in the observed field.

## 4. Conclusions

An integral equation approach is presented to model gain media at optical frequencies. The basic philosophy of the approach is illustrated on an example that involves doubly periodic infinite arrays of gold nanorods under an external illumination. As a starting point, the effect of the active material, such as dye molecules, are represented as equivalent electric current sources proportional to the electric field intensity observed inside the nanorod. The prospective study involves deriving a more accurate relation between the equivalent sources at the frequency of operation and the observed electric field intensity at the pumping frequency.

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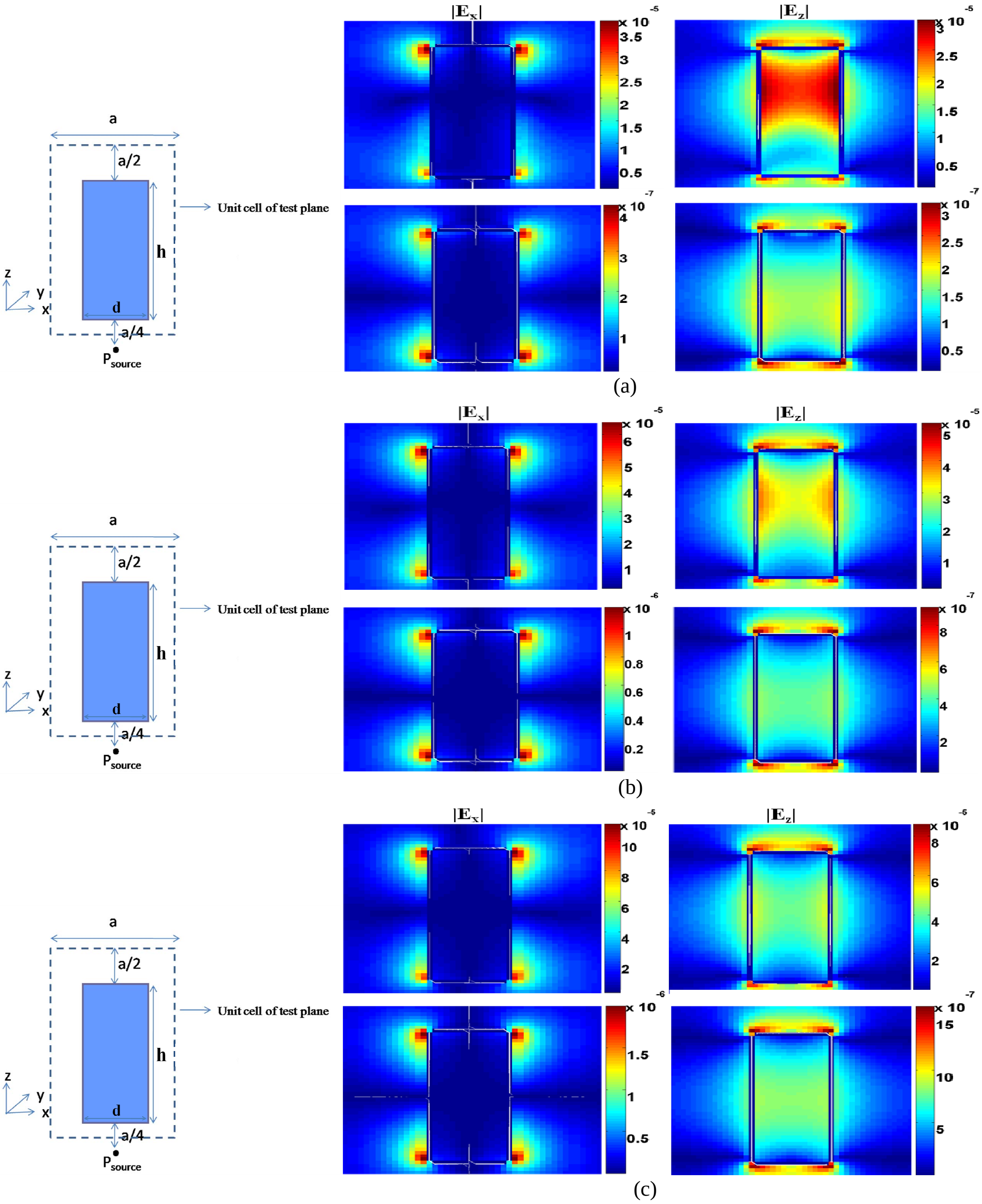


**Fig. 1.** The x and z components of the electric field over a unit cell in the test plane. On the left (a=50 nm, d=20 nm, h=52 nm), $P_{source}$ represents $\hat{z}$ -oriented electric current density of unit amplitude. The x and z components of the electric field at the top correspond to the total electric field intensity due to the original electric point source below the nanorod for (a) $\lambda_o$=520 nm, (b) $\lambda_o$=540 nm, and (c) $\lambda_o$=552 nm. The x and z components of the electric field at the bottom correspond to the radiated electric field due to equivalent sources along the axis of the nanocylinder, that represent the effect of dye molecules inside the nanorod for (a) $\lambda_o$=520 nm, (b) $\lambda_o$=540 nm, and (c) $\lambda_o$=552 nm.

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