Bloch Theory and Metadispersion

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Abstract

In this work, a general and systematic method for the characterization of dispersion in metamaterials realized by stacks of planar periodic surfaces is presented. The analysis procedure subdivides the artificial material into its constituent planes, performs a full wave analysis of the individual planar periodic structures, and uses analytic formulas from Bloch theory to determine the dispersion characteristics of the overall artificial medium. An extension of the conventional Bloch theory to the case of coupled periodically loaded lines is proposed to generalize the analysis to a completely arbitrary propagation vector. Physical properties of the equivalent admittance matrix modeling the single planar sheet are exploited to analytically describe the dispersion properties of the metamaterial in the whole first Brillouin zone starting from a limited number of 2D simulations.

1. Introduction

In the last decade the electromagnetic engineering community has focused its attention on the analysis and modeling of metamaterials [1]. Among the various parameters characterizing a metamaterial, the dispersion diagram provides a clear physical picture for understanding the wave propagation and constitutes a useful basis for the definition of practical design criteria. Dispersion diagrams can be obtained through transmission measurements of a metamaterial sample or 3D simulations of the metamaterial unit cell. However, these approaches are not sufficient to provide a clear insight into the relationship between the geometrical characteristics of the metamaterial and the corresponding physical behavior. The large number of measurements or simulations required for the determination of the whole dispersion diagram is another drawback of these approaches.

In this work, we present a general approach for the efficient analysis of the dispersion properties of artificial materials consisting of stacks of planar periodic structures. This class includes many metamaterials of practical interest, since artificial materials are typically fabricated by stacking planar arrays of patches printed on dielectric substrates [2].

2. Derivation of the Dispersion Characteristics

The analysis procedure starts from the decomposition of the artificial material into its constituent planar structures. The full wave analysis of the individual planar structures is then performed and an equivalent transmission line (TL) model is defined. The assumption is made that the distance between adjacent planar structures is sufficiently large to render the interactions among higher order Floquet modes negligible. As a consequence, the equivalent network for a plane wave impinging on the planar periodic structure must only account for the dominant Floquet mode, and consists of two TLs associated with the transverse electric (TE) and the transverse magnetic (TM) polarizations loaded by a shunt load represented through an equivalent admittance matrix [3]. The three-dimensional metamaterial is therefore modeled through a couple of periodically loaded TL (Fig. 1a). When the direction of incidence is on a principal plane of the planar periodic structure the admittance matrix is diagonal and the two equivalent TLs are decoupled. In this case, the conventional Bloch theory [4] can be applied to study the propagation in the metamaterial. A similar approach has been employed in [5] to study the

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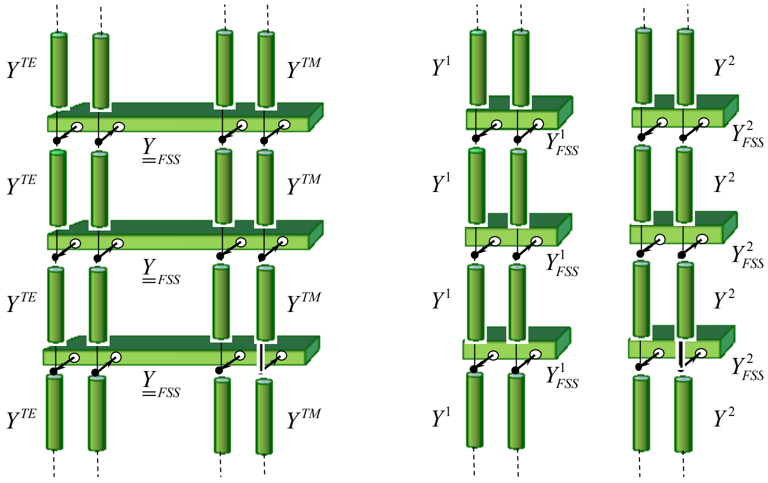
propagation in the wire medium. However, out of the principal planes the application of the conventional Bloch theory would require the solution of a 4x4 eigenvalue equation. In this work, a novel and simpler approach is proposed for the analysis of the periodically loaded coupled lines. It is based on a projection of the normalized modal fields onto the eigenvector basis of the normalized single layer admittance matrix and leads to the definition of a new equivalent TL network in which the two constituent TLs are decoupled (Fig. 1b) and are associated with the actual modes supported by the 3D periodic structure.

All the possible transverse phasings are investigated by considering the single planar periodic structure illuminated by different plane waves (either homogeneous or non homogeneous). Then, the generalized Bloch model is applied to identify two modes supported by the three-dimensional metamaterial. These modes are in general hybrid (*i.e.* neither TE nor TM) and exhibit different longitudinal propagation constants. This means that the metamaterial is in general a birefringent medium, *i.e.* an incident wave impinging on it from free space can give rise to two refracted waves with different ray directions and ray velocities. Furthermore, the Bloch analysis also provides the characteristic impedance of the equivalent TLs (*i.e.* the ratio between the transverse electric and magnetic field at the boundary of the unit cell), which can be used to set equivalent boundary conditions that accurately describe the scattering from a metamaterial slab consisting of a finite number of planar sheets.
The final result of the analysis is, for any given combination of frequency and transverse phasing, a closed-form expression of the corresponding longitudinal propagation constants as a function of the entries of the equivalent admittance matrix of the constituent planar periodic structure.



(a) (b)

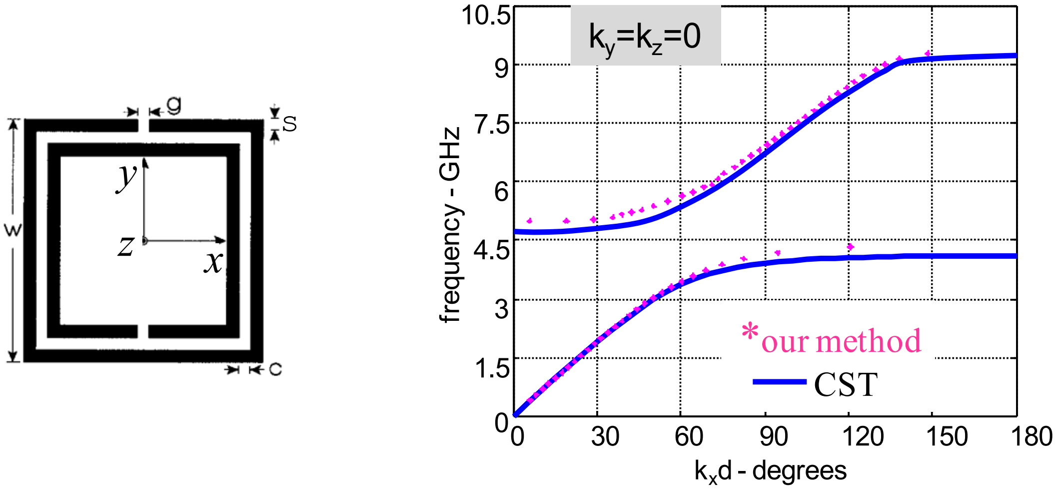
Fig. 1 Equivalent TL model for a generic stack of planar periodic structures: a): classical model with coupled TE and TM equivalent TLs; b): novel model with two decoupled equivalent TLs.

3. Analytical Reconstruction of the Equivalent Admittance Matrix

A key point for the efficiency of the dispersion analysis is the possibility to completely reconstruct the dependence of the equivalent admittance matrix on frequency and transverse phasing starting from a limited number of simulations. Here, this is done by using a pole-residue matching technique similar to the one described in [6]. This technique exploits the Foster's properties of lossless two-port networks to derive an analytical description of the frequency dependence of the entries of the equivalent admittance matrix based on the identification of poles and residues. These parameters exhibit a weak dependence on the transverse phasing, thus, allowing one to apply an efficient interpolation algorithm to obtain the overall response of the periodic sheet from a limited number of full-wave simulations.

4. Numerical Results

The proposed technique has been applied to the analysis of the dispersion relation in a square Split Ring Resonator metamaterial. The unit cell, whose geometry is depicted in Fig. 2(a), consists of a pair of concentric square metallic rings with slits etched in opposite sides [7]. Figures 2(b) shows an example of retrieved dispersion curve of the volumetric metamaterial in the condition $\left(k_y, k_z\right) = \left(0, 0\right)$. This curve was obtained by exploiting the analytical representation of the Bloch propagation constant provided by the pole-residue matching technique to determine the value of the transverse wavenumber k_x corresponding to a vanishing value of the longitudinal propagation constant k_z. The results are compared with a full-wave eigenmode solution from the commercial software CST and a very good agreement is observed between the two techniques.



(a) (b)

Fig. 2 Dispersion in a volumetric SRR metamaterial. (a) Geometry of the unit cell; geometrical parameters: $d_x = d_y = d$=10mm, w=0.8d, s=0.08d, g=c=0.04d. (b) Dispersion curve for the case $\left(k_y, k_z\right) = \left(0, 0\right)$

5. Conclusion

A novel method for the complete analysis of dispersion in a metamaterial consisting of a stack of planar periodic structures has been presented. The proposed approach is systematic and computationally efficient, since it only requires a limited number of 2D full wave analysis of one single periodic sheet.

6. Acknowledgment

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