Analysis of metamaterial slab of magnetodielectric spheres

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Abstract

In this paper we analyze an extremely large, one-layer slab consisting of magnetodielectric spheres. The theory of vector spherical harmonics and the vector addition theorem with infinite array approximations were used to determine the sphere scattering coefficients, and subsequently the homogeneous medium constitutive parameters ϵ_r and μ_r. In contrast to the existing methods based on scattering from a finite array of spheres that require large memory, our method requires very little memory and is more accurate as array size increases, rendering the method applicable to extremely large array problems.

1. Introduction

Recently, metamaterials consisting of magnetodielectric spheres have attracted much attention because of their almost-isotropic and low-loss properties, which make them candidates for many potential applications, e.g. lenses of sub-wavelength resolution. Even though these materials have been analyzed by a number of authors [1,2], it is still not clear how accurate their solutions are since each of them made different approximations in their analyses. For instance, in [2] the authors analyzed a 3-D periodic array of magnetodielectric spheres considering only the fields of the lowest order spherical multipoles. Since the spheres are closely spaced it is of great interest to understand the effects of higher order multipoles on ϵ_r and μ_r. Therefore, we are presenting a method for evaluating the effective constitutive parameters, ϵ_r and μ_r of a 3D array of magnetodielectric spheres where in addition to the lowest order spherical multipoles, higher order spherical multipoles are also included in the analysis.

To accurately determine the effective constitutive parameters of the body of multiple spheres, the number of spheres must be extremely large. Existing codes which simulate the scattered field from a large number of coupled spherical multipoles require prohibitively large computer memory. In our infinite and periodic array method, the scattering coefficients of every sphere are identical and consequently, the method is memory independent. In the numerical calculations the infinite sum over elements is truncated at very large indices where the error due to edge effects is small.

2. Analysis

An infinite 2D array of magnetodielectric spheres is shown in Fig. 1. The sphere radius is a, while the array lattice is rectangular with element spacings, d_x and d_y in the x and y directions, respectively. The elements (spheres) are designated by two indices (p, q) in x and y directions, respectively. We consider a plane wave $\mathbf{E}_i(x, y, z) = \hat{\mathbf{x}}\, exp(ik_0 z)$ incident in $+z$ direction on the "one-layer" metamaterial slab.

ISBN 978-952-67611-0-7

Due to lack of space, below we indicate only the major steps in the analysis. The total incident field on the reference element $(p = 0, q = 0)$ is that of the incident plane wave plus the scattered field from all other elements. Applying the boundary conditions on the sphere yields the system of inhomogeneous equations for the scattering coefficients a^r_{nm}, b^r_{nm}, (n,m) being spherical harmonics indices. The matrix that must be inverted is of $N \times N$ size where $N = n^2_{max} + 2\, n_{max}$ and n_{max} is the maximum number of spherical multipoles. With known a^r_{nm}, b^r_{nm}, we calculated the scattered far-field due to lowest order spherical multipoles (dipoles) by summing over all element contributions ($\sum_{pq}$). Subsequently, the spherical Hankel functions in expressions for the scattered far-field were approximated with their large argument form and then far-field expressions are transformed into Cartesian coordinates yielding the field $\mathbf{E_r}(\mathbf{r}') = \hat{\mathbf{x}} E_x$. To accelerate convergence of the $\sum_{pq}$, using Poisson sum formula, the expression for E_x is transformed into a unit cell (phase-shift-wall waveguide) representation in terms of Floquet modes. In the far field $(k_0 z >> 0)$ only the dominant TEM mode propagates. The problem, thus, is reduced to a single unit cell with the TEM mode incident on a single sphere where reflected (scattered field in -z direction) and transmitted TEM modes (scattered field in +z direction) are known. From here we readily obtain S_{11} and S_{21} at the reference planes $z = -d_z/2$ and $z = d_z/2$, respectively. With these S parameters, using known transmission line theory, we retrieve the effective constitutive parameters ϵ_r and μ_r.

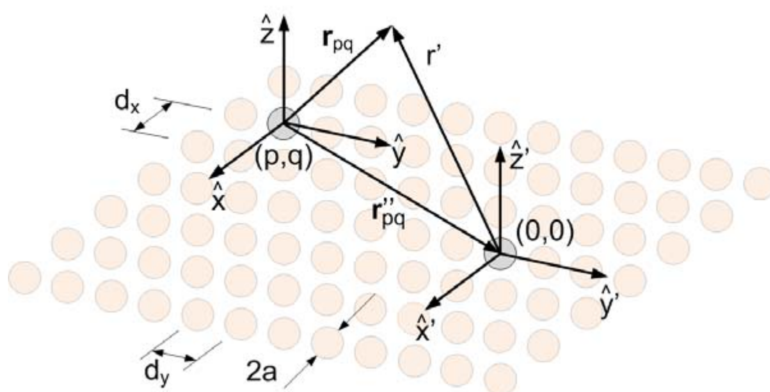

Figure 1: A 2D infinite array of magnetodielectric spheres showing two coordinate systems used in the analysis

3. Results

Preliminary results for scattering coefficients are shown in Fig. 2 at $f_0 = 10\,GHz$ for the following array parameters: sphere radius $a = 0.05\lambda_0$ with relative permittivity and permeability of $\epsilon_r = 13.8,\ \mu_r = 11$, respectively. The element spacings are $d_x = d_y = d_z = a/0.45$. The red and blue curves in the top two figures show the scattering coefficients of the lowest order spherical multipoles (dipoles) a^r_{11} and b^r_{11} for a linear array of 101 spheres placed along the x-axis. The coefficients were calculate by an in-house developed code for the scattered field from a finite number of spheres [3]. Notice the highly oscillatory behavior of both curves due to the edge effects. The black line is the equivalent infinite array solution obtained by summing 20,001 terms in $\sum_{pq}$. As expected, the infinite array data is an average of the finite array values. Similarly, the bottom two curves compare the a^r_{11} and b^r_{11} of the same array along the y-axis with the infinite array data. Again, the infinite array solution is an average of highly oscillatory finite array values. It is interesting to note that the oscillations are much more pronounced in the array along the y-axis (H-plane) since the coupling between the spheres is higher than in the E-plane for x-directed incident field. Consequently the edge effects are much more severe in the array along the y-axis. From this data it is obvious that finite arrays of magnetodielectric spheres can't be used to accurately

ISBN 978-952-67611-0-7

characterize the homogeneous body effective constitutive parameters. For clarity, here we only present results for the expansion coefficients a^r_{11} and b^r_{11} of x and y-directed linear arrays. 2D arrays have been analyzed and the corresponding results for a^r_{11}, b^r_{11} and the retrieved constitutive parameters ϵ_r and μ_r will be presented at the conference.

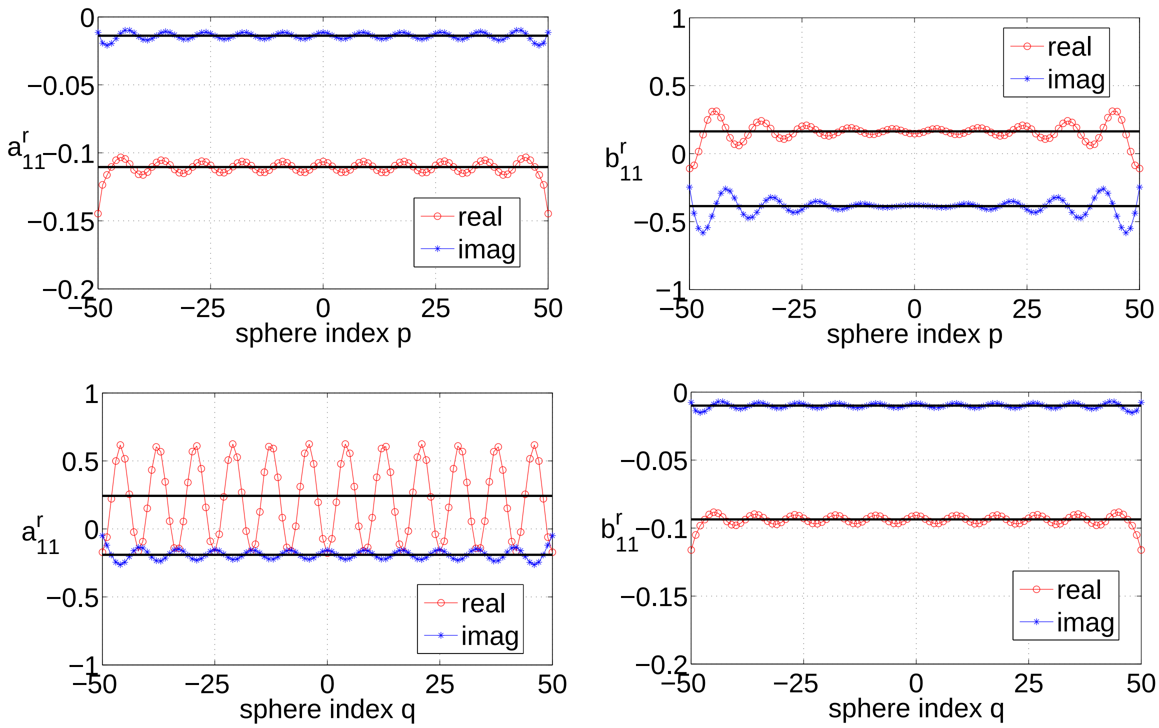

Figure 2: Scattering coefficients a^r_{11} and b^r_{11} for linear array of spheres along x-axis (top two figures) and along y-axis (bottom two figures), red and blue markers=finite array, black line=infinite array

4. Conclusion

We have developed a method for the determination of effective constitutive parameters ϵ_r and μ_r of an array of magnetodielectric spheres. To our knowledge, this is the first time higher order spherical multipoles have been included in the analysis. We have shown that finite arrays of magnetodielectric spheres exhibit high edge effects and therefore can't be used for accurate constitutive parameter extraction. The method requires very little memory and is more accurate as array size increases, rendering the method applicable to extremely large array problems.

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