Magnifying absolute instruments for homogeneous regions

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Abstract

We propose several magnifying absolute optical instruments that create magnified stigmatic images, either virtual or real, of optically homogeneous three-dimensional spatial regions.

1. Introduction

Most optical instruments have various types of aberrations. There exist, however, so-called absolute optical instruments that are free of aberrations and provide sharp images of all points in some 3D region of space within geometrical optics [1]. A prototype of an absolute instrument is Maxwell's fish eye, a device designed in 1854 by J. C. Maxwell [2]. It uses positive refractive index and images stigmatically the whole space.

Among absolute instruments there is a class of a particular interest, namely devices whose object and image spaces are optically homogeneous regions, i.e., regions with a uniform refractive index. Until recently, the only known such devices were plane mirrors [1]. However, since the work [3] we know that some well-known optical devices such as Eaton lens or Luneburg lens [4] are in fact absolute instruments as well. All of these devices have unit magnification, giving an image of the same size as the original object, and no magnifying absolute instrument for homogeneous regions has been known. Here we present the first several magnifying absolute instruments.

2. The devices

Our first device has two distinct regions (see Fig. 1 (a)). Region I is a sphere of unit radius, region II is the space between two hemispheres with the radii 1 and $R > 1$, lying in the half-space $y > 0$. Both regions
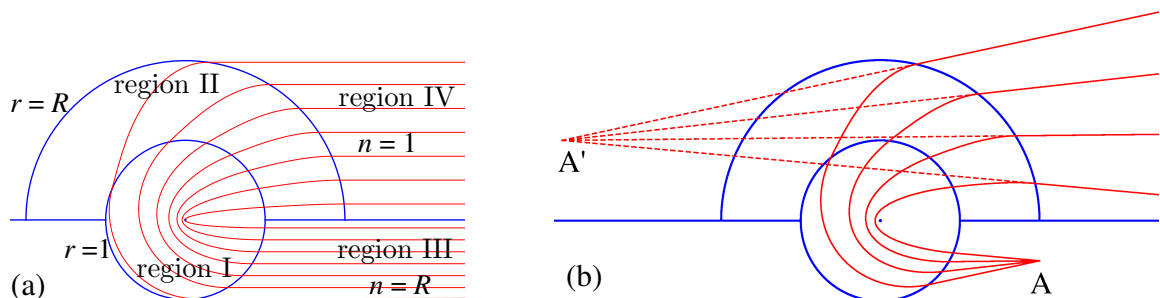

Fig. 1: Magnifying Eaton lens with $R = 2$. (a) Light rays incident from region IV are changed into rays in region III propagating in the opposite direction, with impact parameters reduced by the factor of R. (b) Light rays emerging from a point A at $\vec{r}_A$ in region III, after passing through the lens, propagate in region IV as if they originated from a point A' at $\vec{r}_{A'} = -R\vec{r}_A$.

are filled with a spherically symmetric refractive index $n_I(r)$ and $n_{II}(r)$, respectively. The indices are chosen such that $n_I(1) = n_{II}(1) = R$ and $n_{II}(R) = 1$. The medium surrounding the lens is composed of two parts as well. In the region $r \geq 1, y < 0$ (region III) the refractive index is R while in the region $r \geq R, y > 0$ (region IV) the refractive index is unity. Thus the index of the lens matches that of the surrounding medium and also the indexes at the border between regions I and II match each other.

The refractive index $n_{II}(r)$ is designed such that a light ray incident from region IV to region II is bent towards the center, and cross the border between regions II and I. There are many index profiles that can do that, one option is $n_{II}(r) = [1 + c(r-1)(R-r)]R/r$ with $c > 0$, which we have also used here with $c = 1$. The refractive index $n_I(r)$ is then designed such that the light ray coming from region II leaves region I for region III in the opposite direction than was the original direction of the ray in region IV, see Fig. 1 (a). We can therefore call the device "magnifying Eaton lens" due to its apparent similarity with Eaton lens.
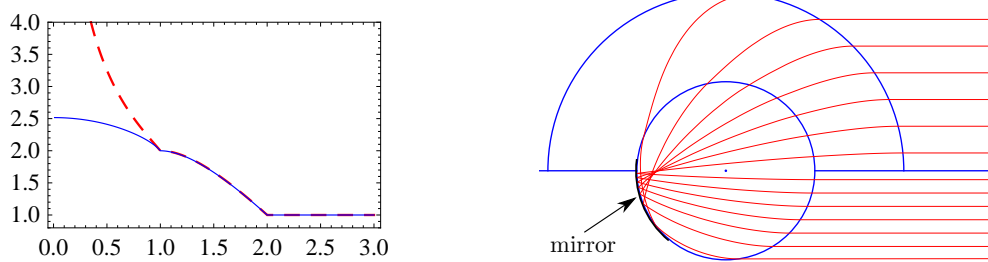

Fig. 2: (a) Refractive index in magnifying Eaton (dashed red) and Luneburg (solid blue) lens with $R = 2$. (b) Magnifying Luneburg lens. A spherical mirror covers part of the interface between regions I and II.

To find $n_I(r)$, we solve the inversion problem [4]. We characterize rays in the lens by the angular momentum $L = nr \sin\beta$, where β is the angle between the radius vector and the ray. Angular momentum is conserved and motion of a particle is planar in central potentials, and the same holds for light ray in a spherically symmetric refractive index. Consider a ray propagating in the plane xy horizontally in region IV. The polar angle h swept by the ray in region II before entering region I is [5]

$$h(L) = L \int_1^R \frac{dr}{r\sqrt{n_{II}^2 r^2 - L^2}}. \tag{1}$$

The scattering angle χ corresponding to motion in region I is therefore $\chi(L) = \pi - h(L)$, which ensures that the total change of ray direction during motion in region II and I is π. Solving the inversion problem, we arrive at [4]:

$$n_I(r) = \exp\left[\frac{1}{\pi}\int_{rn_I(r)}^R \frac{\chi(L)dL}{\sqrt{L^2 - [rn_I(r)]^2}}\right]. \tag{2}$$

From conservation of L it then follows that the impact parameter of the outgoing ray in region II is R-times smaller than the impact parameter of the incoming ray in region IV. Consider rays emerging from a point A at $\vec{r}_A$ in region III and incident on region I. These rays will be transformed by the lens into rays propagating in region IV, each parallel to the original ray in region III. Therefore the lines on which these outgoing rays lie intersect at the point A′ with radius vector $\vec{r}_{A'} = -R\vec{r}_A$, which this way becomes the virtual image of the point A (see Fig. 1 (b)), and the magnification of the device is equal to R. The refractive index in regions I + II is shown in Fig. 2 (a).

The same idea can be now applied to designing other similar magnifying absolute instruments, for example a device resembling Luneburg lens [4]. This way one avoids the singularity of the refractive index at the origin and obtains the lens shown in Fig. 2 (b) with refractive index shown in Fig. 2 (a). Since

the ratio of the largest and smallest refractive index is just slightly larger than R, it should be possible to construct this lens for near infrared or even visible light using e.g. graded index structures in silicon [6] or diamond.
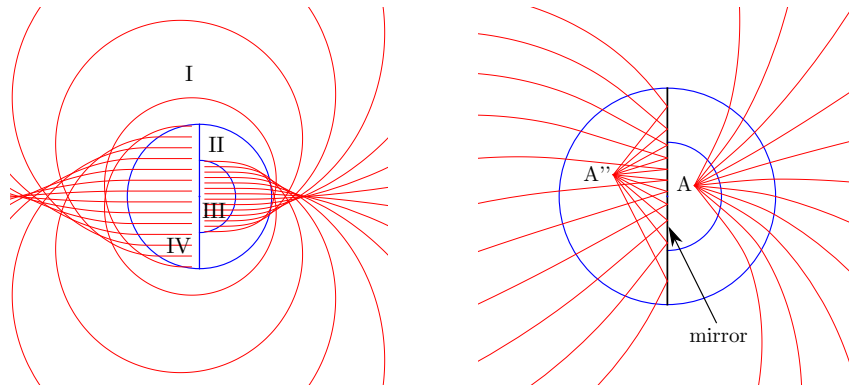

Fig. 3: (a) Magnifying instrument giving a real image. The rays enter the lens from region III and after making loops in region I they enter region IV in their original direction. (b) Rays originating at point A converge in region IV to a point outside this region and are reflected by the mirror to the real image A'' of A.

We can also design is an absolute instrument that provides real images. Region I now occupies the whole 3D space with the exception of the sphere of radius R, see Fig. 3. Light rays enter the lens from the inside. After leaving region I, they enter region IV parallel to their original direction. Consider rays emerging from some point A at radius vector $\vec{r}_A$ in region III, see Fig. 3 (b). The rays get to region IV assuming their original direction and head towards the point A' at $\vec{r}_{A'} = R\vec{r}_A$. This image would be virtual, but we can make it real by employing a mirror at the flat interface of region IV.

3. Conclusion

In conclusion, we have proposed several absolute optical instruments that create magnified stigmatic images of homogeneous 3D region. Especially appealing is the magnifying Luneburg lens with its moderate refractive index range and the lens giving a real magnified image. Magnifying absolute instruments could find their applications in various fields, for example in photolitography, but more importantly, our research has shown that such devices exist at all, something that was not at all clear up to this date [1, 3].

References

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