Cloaking a 3-d arbitrary shaped domain

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Abstract

In this presentation we will describe a straightforward method to design the electromagnetic cloak of a 3d-generic star domain whose surface is defined just by a set of points distributed over it.

1. Introduction

Transformation techniques make it possible to design the distribution of the dielectric permittivity and magnetic permeability of an hollow shell to make it able to exclude every electromagnetic field from its cavity [1], [2]. This topic attracted much attention in last years both from theoretical and experimental point of view. Furthermore, the possibility to design cloaks of arbitrary shapes has been analyzed [3], [4], [6], [7], [8] a straightforward general method to design the cloak of a 3d generic objects remains an unsolved problem.

The transformation technique [1], [2] requires (i) to choose a suitable external surface of the cloak and (ii) to find a spatial transformation able to compress all the space in the volume inside the external surface of the cloak into the shell contained between the internal and the external surfaces (respectively a and b).

We will show a procedure to calculate these quantities starting from a set of geometric points distributed on the surface of the object to be cloaked.

2. Definig and reconstructing the inner surface

Let us suppose, that the surface of the object a is identified through a set A of N points uniformly distributed on it: $A = \{P_1 \equiv (x_1^{(1)}, x_2^{(1)}, x_3^{(1)}), P_2 \equiv (x_1^{(2)}, x_2^{(2)}, x_3^{(2)}), \ldots, P_N \equiv (x_1^{(N)}, x_2^{(N)}, x_3^{(N)})\}$ as in Fig. 1(a). In principle, this A-set is referred to a generic system of coordinates (SOC). We can then calculate the barycenter $B \equiv (x_1^{(B)}, x_2^{(B)}, x_3^{(B)})$ of the A-set of points and make it the origin of a new SOC, with respect to which which we can recalculate the coordinates of points belonging the A-set. As we shall see in the following, we need this change of coordinates because, in order to perform the proper transformation, the more natural way is to realize that in a SOC whose origin is contained inside the object we want to cloak. Let us now calculate for each point of the A-set ($P_n \equiv (x_1^{(n)}, x_2^{(n)}, x_3^{(n)})$ its spherical coordinates $P_n \equiv (\varphi^{(n)}, \theta^{(n)}, r^{(n)})$). It is then possible to define the discrete function $a_n(\varphi^{(n)}, \theta^{(n)}) = r^{(n)}$. The function a_n corresponding to the set of points presented in Fig 1(a) is plotted in Fig. 1(b) as an example. The continuous function $a(\varphi, \theta)$ can obviously be calculated for each value of θ and φ through interpolation of the discrete function $a_n(\varphi^{(n)}, \theta^{(n)})$. Actually we used a linear Delaunay

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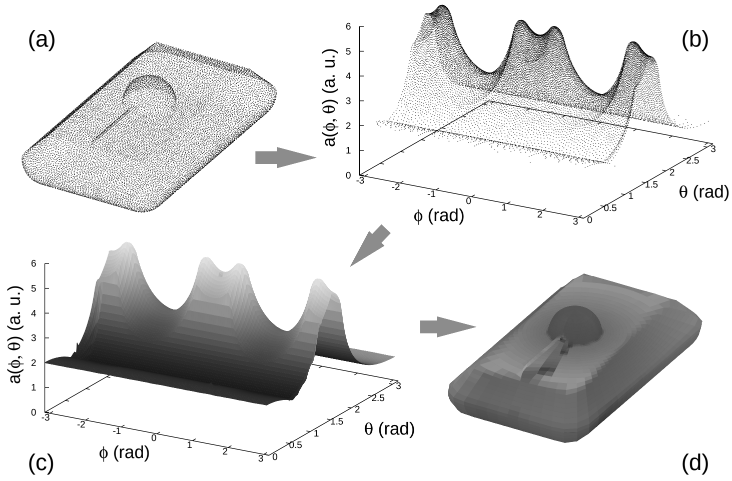

Fig. 1: (a) a set of points distributed on the object to cloak; (b) the discrete polar function $a_i(\varphi_i, \theta_i)$ corresponding to the set of points in (a); (c) the continuous function $a(\varphi, \theta)$ obtained via interpolation from the discrete function $a_i(\varphi_i, \theta_i)$ presented in (b); (d) The object is smoothly reconstructed by reverting function $a(\varphi, \theta)$ of sketch (c) in the Cartesian space.

interpolation algorithm provided by the Open Source computational geometry package CGAL [9]. The equation $r = a(\varphi, \theta)$ finally allows to fully reconstruct the inner surface of the cloak (an example corresponding to the set sketched in Fig 1(a) is presented in Fig 1(d))

3. Transformation

It is necessary to define a coordinate transformation designed to compress all the space inside an outer cloak surface, suitably chosen, defined by $r = b(\varphi, \theta)$ into the space in between the inner cloak surface and this outer cloak surface. As the origin of the SOC is now located in the barycenter of the A-set of points while we choose as the outer surface a simple expansion of the inner one $b(\varphi, \theta) = Sa(\varphi, \theta)$, with $S > 1$. If the domain we want to cloak is or can be reduced to a 3d-star domain (which means any set A in Euclidean space $\mathbf{R}^3$ if there exists a point P_0 such that for all points P_i in A the line segment from P_0 to P_i is in A), we can define an independent transformation for each couple of values of φ and θ, by compressing the interval $0 \leq r \leq b(\varphi, \theta)$ into the interval $a(\varphi, \theta) \leq r \leq b(\varphi, \theta)$. The simpler transformation of this type is a linear one ($r' = \alpha r + a(\varphi, \theta) \qquad i = 1, 2, 3$) with $\alpha = S - 1/S$. From this expression recasted to its Cartesian form by introducing a vector function $\mathbf{a}(\varphi, \theta)$, the following transformation matrix can be derived:

$$\Lambda^i_j = \frac{\partial x'_i}{\partial x_j} = \alpha \delta_{ij} + \frac{\partial a_i}{\partial x_j} \tag{1}$$

and by applying this result to the Cartesian metrics g^{kl} we can calculate the metrics of this new deformed space:

$$g'^{ij} = \Lambda^i_k g^{kl} \Lambda^j_l = \Lambda^i_k \delta^{kl} \Lambda^j_l = \Lambda^i_l \Lambda^j_l = \Lambda^2 \tag{2}$$

which acts on light in a way equivalent to a dielectric tensor calculated as:

$$\varepsilon'^{ij} = \left|\det(\Lambda'^i_j)\right|^{-1} g'^{ij} \varepsilon_0 \tag{3}$$

where ε_0 is the dielectric permittivity in free space. Magnetic permeability can be derived by applying the same transformation to free space so ε'^{ij} and μ'^{ij} are in fact the same symmetric tensor. In Fig. 2

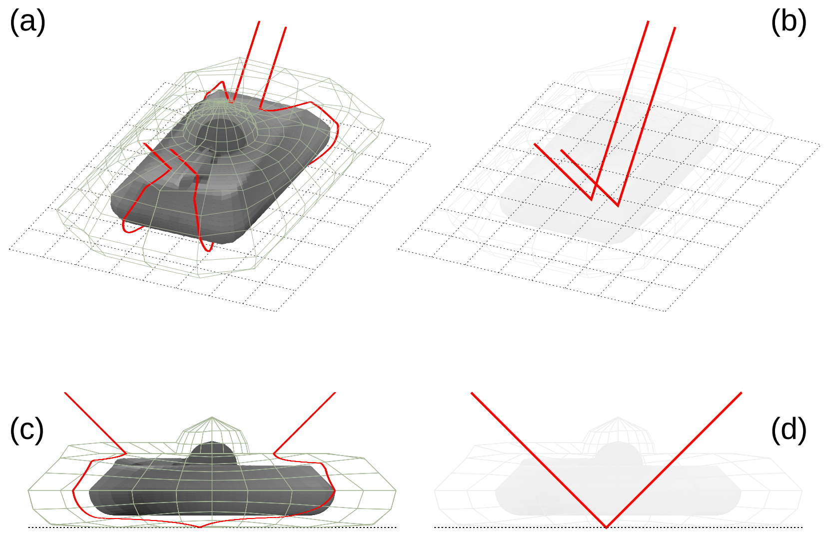

Fig. 2: (a) and (c): Ray tracing simulation for the object described in fig. 1 cloaked using $S = 1.5$ and put on a reflecting plane. (b) and (d): View of the ray tracing on a empty surface plotted as a reference

a ray-tracing simulation is presented to verify the validity of the proposed algorithm. The extended calculations can be found in [10]

4. Conclusion

We presented a straightforward method to design the electromagnetic cloak of a 3d-generic shaped object spatially defined by a set of points distributed over it. This approach can be used on any three-dimensional object realized with commercial or free 3d-design software by acquiring the mesh nodes position as the set of point needed by our method.

References

[1] J. B. Pendry, D. Schurig, and D. R. Smith. "Controlling Electromagnetic Fields". Science, **312**(5781) 1780-1782, (2006).

[2] D. Schurig, J. B. Pendry, and D. R. Smith. "Calculation of material properties and ray tracing in transformation media". Opt. Express, **14**(21) 9794-9804, (2006).

[3] J. Hu, X. Zhou, and G. Hu. "Design method for electromagnetic cloak with arbitrary shapes based on Laplace's equation". Opt. Express, **17**(3) 1308-1320, (2009).

[4] H. Ma, S. Qu, Z. Xu, and J. Wang. "Numerical method for designing approximate cloaks with arbitrary shapes". Physical Review E (Statistical, Nonlinear, and Soft Matter Physics), **78**(3) 036608, (2008).

[5] H. Ma, S. Qu, Z. Xu, and J. Wang. "Approximation approach of designing practical cloaks with arbitrary shapes". Opt. Express, **16**(20) 15449-15454, (2008).

[6] W. X. Jiang, J. Y. Chin, Z. Li, Q. Cheng, R. Liu, and T. J. Cui. "Analytical design of conformally invisible cloaks for arbitrarily shaped objects". Physical Review E (Statistical, Nonlinear, and Soft Matter Physics), **77**(6) 066607, (2008).

[7] J. Zhang, Y. Luo, H. Chen, and B. I. Wu. "Cloak of arbitrary shape". J. Opt. Soc. Am. B, **25**(11) 1776-1779, (2008).

[8] W. Yan, M. Yan, Z. Ruan, and M. Qiu. "Coordinate transformation makes perfect invisibility cloak with arbitrary shape". New J. Phys., **10**, (2008).

[9] CGAL, Computational Geometry Algorithms Library. http://www.cgal.org.

[10] A. Veltri. "Designs for electromagnetic cloaking a three-dimensional arbitrary shaped star-domain". Opt. Express, **17**, (2009).