Bloch Modes of Optical Fishnet Metamaterials: a "Microscopic" Model

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Abstract

We theoretically study fishnet metamaterials at optical frequencies. In contrast to earlier works, we provide a microscopic description by tracking the transversal and longitudinal flows of energy through the fishnet mesh composed of intersecting subwavelength plasmonic waveguides. The analysis is supported by a semi-analytical model based on surface-plasmon coupled-mode equations. The model provides accurate formulas for the fishnet refractive index, including the real-negative and imaginary parts. The model simply explains how the surface plasmons couple at the waveguide intersections and it shines new light on the fishnet negative-index paradigm at optical frequencies. Extension of the theory for loss-compensated metamaterials with gain media is also presented.

1. Introduction

In the recently emerged fields of metamaterials and transformation optics , the possibility of creating negative-index metamaterials (NIMs) using nanostructured metal-dielectric composites has spurred extensive research over the past few years [1-2]. Until now, our common perception of optical NIMs is entirely based on the concept of homogenization. Different techniques have been developed, all of them being based on fully-vectorial electromagnetic calculations of the whole structure. As a consequence, even if these approaches quantitatively predict the light transport, they are often non-intuitive, thereby hindering the design process required to apply metamaterials into new optical technologies. For a better understanding, we study fishnet NIMs [3-4] by adopting a "microscopic" point of view inspired by a similar approach which has been applied to explain extraordinary optical transmission successfully [5]. We abandon classical homogenization approaches and instead we track the energy as it propagates and scatters through the fishnet mesh, like a fluid flowing in a multi-channel crossed system. For 3D fishnet NIMs, this formalism provides closed-form expressions for the effective index and, due to an analytical handling of the key parameters, it shines new light on the physical origin of negative refraction in fishnets.

2. Description of the "Microscopic" Model and Results

A fishnet NIM consists of an array of subwavelength air holes drilled into a metal-insulator-metal (MIM) periodic stack. It is worth emphasizing that all the prominent optical properties of a fishnet are driven by its fundamental Bloch mode. This statement was firstly observed in the experiment [3], where the measured refractive index is equal to the effective index n_{eff} of the fundamental Bloch mode, and has been recently confirmed by numerical results showing that energy transport inside the fishnet is solely mediated by the fundamental Bloch mode [4]. Therefore, our analysis is mainly devoted to understanding the origin of the negative values of the effective index of the fundamental Bloch mode.

The model involves two intersecting subwavelength channels and their coupling, see Fig. 1. The vertical (z-direction) channel is composed of a one-dimensional (1D) hole chain in a metal film; and we

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assume that the vertical energy transport is only driven by a single super-mode, which is formed by the in-phase superposition of the fundamental TE_{01} mode of every hole. In the horizontal (*x*-direction) channel, which is formed by metal-insulator-metal (MIM) waveguide, we also assume that only a single gap-SPP mode with symmetric transverse magnetic field mediates the light transport. Both channels, along with the definition of the associated scattering coefficients, are depicted in Fig. 1. At the hole chain boundaries, the incident super-mode is partly transmitted (coefficient τ), reflected (coefficient ρ) or scattered (coefficient α) into the gap-SPP mode of the transversal MIM channel, see Fig. 1(b). Similarly, an incident gap-SPP is scattered by the hole chain, see Fig. 1(c). Because of Lorentz reciprocity theorem, the horizontal scattering process only defines two additional coefficients denoted by r_{sp} and t_{sp}. All the scattering coefficients, τ, ρ, α, r_{sp} and t_{sp} have been calculated by using a fully-vectorial frequency-domain modal method [6].

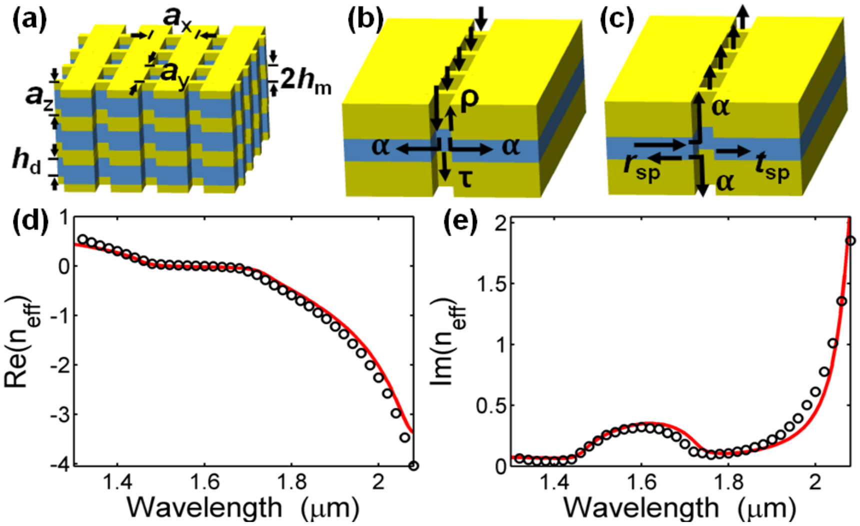

Figure 1. Elementary scattering events involved in a fishnet. **(a)** The fishnet under study is a two-dimensional array (period $a_x = a_y = 860$ nm) of rectangular holes (width $w_x = 295$ nm and $w_y = 595$ nm) etched into a Ag($h_m = 50$ nm)-MgF_2($h_d = 50$ nm)-Ag($h_m = 50$ nm) periodic stack. The refractive index of MgF_2 is $n_d = 1.38$ and the permittivity of silver is taken from [7]. **(b)** Scattering of an incident super-mode of a 1D hole chain. **(c)** Scattering of an incident gap-SPP mode supported by a MIM waveguide. Real **(d)** and imaginary **(e)** part of the effective index of the fundamental Bloch mode supported by the fishnet. The microscopic model predictions are shown by the red solid curve and fully-vectorial calculations are shown by black circles.

Using all these elementary scattering coefficients, we derived a set of coupled-mode equations depicting how both channels exchange energy, which finally leads to a semi-analytical formula of the effective index of the fundamental Bloch mode of the fishnet:

$$\cos(k_0\, n_{\mathrm{eff}}\, a_z) = \frac{(\tau+\gamma)^2 v^2 - (\rho+\gamma)^2 v^2 + 1}{2(\tau+\gamma)v} \tag{1}$$

where $\gamma = 2\alpha^2 u/[1 - (t_{sp} + r_{sp})u]$. $u = \exp(ik_0 n_{sp} a_x)$ is the gap-SPP phase-delay over one period, and n_{sp} is the normalized propagation constant of the gap-SPP. Mathematically, the transversal coupling between the hole chains is fully described by a single parameter γ, which physically represents the multiple scattering of the gap-SPP in the transversal MIM layers.

The model predictions for the fishnet effective index are shown with the solid red curves in Figs. 1(d) and (e). They are found to quantitatively capture all the major features of the calculated data (circles), such as the broadband negative Re(n_{eff}), and the low-loss band of Im(n_{eff}) for $1.8 < \lambda < 2$ µm followed by a rapid increase of the loss at longer wavelengths. Even the band-gap of Im(n_{eff}) for 1.45 µm< λ < 1.75 µm is accurately predicted.

Reducing the attenuation of optical NIMs is crucial and the issue of loss compensation with gain media has recently received much attention [8-9]. Our model allows the analysis of gain-assisted fishnet NIMs. The loss in fishnet is mainly due to the dissipation of gap-SPP in horizontal channels. In the low-loss band, 1.75 µm < λ < 1.95 µm, we assume that the amplification process can be simply analyzed by a phenomenological MgF_2 refractive index $n_d = 1.38 - ig$ with a constant negative imaginary part in order to compensate the loss of gap-SPP. We find that the attenuation Im(n_{eff}) is significantly lowered as g increases, see Fig. 2(a). For $g = 0.022$, Eq. (1) predicts that the fishnet becomes an amplifying medium [Im(n_{eff}) < 0] for 1.75 µm < λ < 1.85 µm in agreement with fully-vectorial calculations, as shown by the black curve in Fig. 2(a). Figure 2(b) compares the transmission T obtained either by the Fabry-Perot model assuming that the energy transport through the fishnet slab is solely mediated by the fundamental Bloch mode [4] or by fully-vectorial calculations obtained with the Rigorous-Coupled-Wave-Analysis.

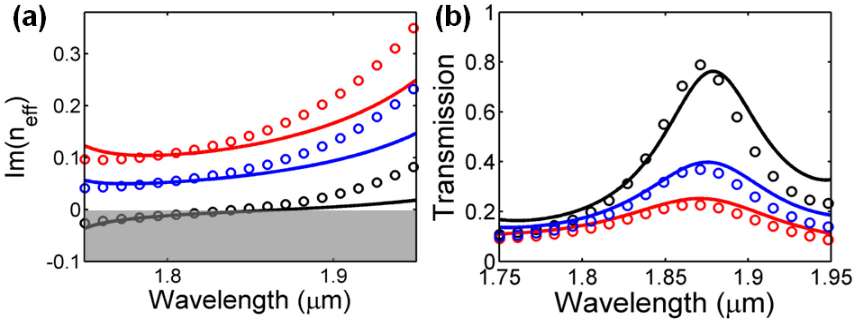

Figure 2. Loss compensation with gain. **(a)** Fishnet attenuation Im(n_{eff}) for three gain values, $g = 0$ (red), 0.01 (blue) and 0.022 (black). Re(n_{eff}) is nearly indepedent of g. **(b)** Transmission spectra of a finite-thickness ($d = 5a_z$) fishnet for the same g's values. In **(a)** and **(b)**, the model predictions are shown with solid curves and fully-vectorial calculations are shown with circles. For the sake of clarity, in **(b)** the model predictions are blue-shifted by 20 nm to compensate for the slight offset in Re(n_{eff}) due to the small metal thickness.

3. Conclusion

In summary, light transport in fishnet NIMs at optical frequencies has been analyzed with a comprehensive and accurate model. In addition to providing the first analytical treatment of NIMs at optical frequencies, the model shines new light on how a negative index is formed and how the inevitable losses associated to a magnetic-like resonance can be compensated by incorporating gain. We hope that our contribution may be helpful, not only to further designs of NIMs at optical frequencies, but also to engineer complex metallo-dielectric surfaces in general.

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