Biaxial negative magnetic permeability support magnetic surface polariton mode

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Abstract

In this study, we provide a dispersion equation of magnetic surface polariton (MSP) mode supported by an anisotropic magnetic material (AMM). According to the dispersion equation, it requires controlling three frequency-dependent constituent parameters properly, so that then we design a biaxial magnetic metamaterial (PBMM) accordingly. Eventually, the realization of the MSP mode under transverse electric excitation by the designed PBMM is evidenced by both analytical calculation and numerical simulation.

1. Introduction

The electric surface plasmon polariton (ESPP) mode has been increasingly studied, in particular for the applications of biosensing [1] and nanophotonics [2]. Analogous to the ESPP mode, the very rare phenomenon of magnetic surface polariton (MSP) mode has first demonstrated by using an uniaxial antiferromagnetic (FeF₂) at far-infrared region [3]. In general, both the ESPP and MSP modes can be realized by natural materials, but deficient in their electromagnetic response due to the limited frequency-dependenct constituent parameters. Recently, a new class of artificially constructed materials termed as metamaterials, promise to extend the limited constituent parameters to enable a variety of unprecedented electromagnetic. Yet, metamaterials are usually anisotropic, so that the conventional isotropic dispersion equation of the ESPP [4] and MSP mode [3] need to be further revised to be applied on the anisotropic metamaterials.

2. Theory

We first start from considering two homogeneous media, a free space in the upper space (y > 0) and an AMM in the lower half space (y < 0), and assume that the optic axes of the AMM coincide with the Cartesian coordinate [see Fig. 1(a)]. Without loss of generality, the electric permittivity ($\varepsilon_{air} = 1$) and magnetic permeability ($\mu_{air} = 1$) are for free space; meanwhile, the effective electric permittivity tensor (ε_{eff}) and the effective magnetic permeability tensor (μ_{eff}) are for the AMM [see Eq. (1)]. For the purpose of a simplified discussion, we consider these two tensors real.

$$\varepsilon_{eff}(\omega) = \begin{pmatrix} \varepsilon_x & 0 & 0\\ 0 & \varepsilon_y & 0\\ 0 & 0 & \varepsilon_z \end{pmatrix}, \mu_{eff}(\omega) = \begin{pmatrix} \mu_x & 0 & 0\\ 0 & \mu_y & 0\\ 0 & 0 & \mu_z \end{pmatrix}$$
(1)

The anisotropic dispersion equation of the MSP is calculated as follows. Whenever the MSP mode is excited, we expect that the electric field of the MSP mode can has a z-polarization with exponentially decay along both the $\pm y$ directions. Therefore, we have $\vec{E}^+(\omega) = \hat{z}\tilde{E}_z^+e^{i(k_xx)}e^{-\alpha^+y}$ for air

and $\vec{E}^{-}(\omega) = \hat{z}\tilde{E}_{z}^{-}e^{i(k_{x}x)}e^{\alpha^{-}y}$ for the AMM, where \tilde{E}_{z}^{\pm} and α^{\pm} are complex amplitude and attenuation coefficients, respectively. Then, according to Faraday's law, we obtain the magnetic flux of the MSP mode are $\vec{B}^{+}(\omega) = (\frac{c}{i\omega})\left[(-\alpha^{+})\hat{x}-(ik_{x})\hat{y}\right] \cdot \tilde{E}_{z}^{+}(\omega)e^{i(k_{x}x)}e^{-\alpha^{+}y}$ for air and $\vec{B}^{-}(\omega) = (\frac{c}{i\omega})\left[(\alpha^{-})\hat{x}-(ik_{x})\hat{y}\right] \cdot \tilde{E}_{z}^{-}(\omega)e^{i(k_{x}x)}e^{\alpha^{-}y}$ for the AMM. Moreover, we also assume the AMM is a linear medium, so that we have $\vec{D}(\omega) = \varepsilon \vec{E}(\omega)$ and $\vec{B}(\omega) = \mu \vec{H}(\omega)$. Then, we take Ampere's law so that we have attenuation coefficients, $\alpha^{+} = \sqrt{k_{x}^{2} - \frac{\omega^{2}}{c^{2}}}$ and $\alpha^{-} = \sqrt{\frac{\mu_{x}}{\mu_{y}}k_{x}^{2} - \frac{\mu_{x}}{c^{2}}\varepsilon_{z}}$. The attenuation coefficients should be real to ensure that the MSP decays exponentially away from the interface. On the other hand, the tangential components of the electric field (E_{z}) and the magnetic field (H_{x}) at the interface (y = 0) have to be continuous, yielding to $\vec{B}_{x}^{+} = (1/\mu_{x})\vec{B}_{x}^{-}$. Eventually, we obtain the dispersion equation of the MSP mode [see Eq. (2)].

$$k_x = \frac{\omega}{c} \sqrt{\frac{\mu_y(\mu_x - \varepsilon_z)}{\mu_y\mu_x - 1}} = k_{MSP} = \frac{2\pi}{\lambda_{MSP}}$$
(2)

As a consequence, to support the MSP mode we necessitate real numbers of k_{MSP} and α^{\pm} , which can be fulfilled by the requirements of $\mu_x < 0 < \varepsilon_z$ and $\mu_x \mu_y > 1$.



Fig. 1: Structure diagram and unit cell of PBMM. (a) y > 0: free space; y < 0: the AMM. The xy plane is an incident plane. (b) The unit cell of the designed PBMM composes of a low loss Roger RT5800 board which sandwiched by two artificial layers, one-cut SRRs shown in (c) and three-cut SRRs shown in (d). Dimension: $a_x = 6 \text{ mm}$; $a_y = 2 \text{ mm}$; $a_y = 6 \text{ mm}$; $g_1 = 0.25 \text{ mm}$; $s_{z1} = 5.75 \text{ mm}$; $s_{x1} = 5.85 \text{ mm}$; $w_1 = 0.20 \text{ mm}$; $g_{2-1} = 0.30 \text{ mm}$; $g_{2-2} = 0.28 \text{ mm}$; $g_{2-3} = 0.30 \text{ mm}$; $s_{x2} = 5.75 \text{ mm}$; $s_{z2} = 4.40 \text{ mm}$; $w_2 = 0.2 \text{ mm}$. The thickness of the substrate and the SRR are 0.254 mm and $1.7 \times 10^{-2} \text{ mm}$, respectively.

3. Simulation

Here we design a planar biaxial magnetic metamaterial (PBMM) [y < 0, see Fig. 1(a)], to substantiate the MSP mode. The unit cell of the designed PBMM [see Fig. 1(b)] is comprised of two distinct SRRs made by perfect electric conductors [see in Fig. 1(c) and 1(d)] with a low loss substrate of Roger RT5880. The retrieved methods [5, 6] show that this planar structure presents $\mu_x = -4.20$, $\mu_y = -0.252$ and $\varepsilon_z = 3.46$ at 13.22 GHz, which fulfill the requirements of $\mu_x < 0 < \varepsilon_z$ and $\mu_x \mu_y > 1$ aforementioned to enable the MSP mode. Next, we employ a commercial electromagnetic solver, COMSOL, to scrutinize the MSP mode supported by the PBMM, in which we set diffraction gratings as an optical coupler and a normally TE Gaussian beam with z-polarization as the incident wave [see Fig. 2(a)]. From Fig. 2(a), it manifests that there exists a wave propagating along the interface with exponentially decay. The nonradiative nature of this MSP mode is evidenced by the exponentially decaying electric and magnetic field along the y direction at x = 100 mm [see Figs. 2(b) and 2(c)] and furthermore, the average wavelength ($\overline{\lambda}_{MSP}$) of this MSP approximates 3.940 mm [see the inset of Fig.2], which nearly corresponds with the wavelength of the MSP mode predicated by Eq. (2) ($\overline{\lambda}_{MSP} = 3.947$ mm). The analytical calculation and numerical simulation unambiguously demonstrates that one can realize the MSP mode of the PBMM under TE excitation.



Fig. 2: Distributed y-component of the magnetic fields of the MSP mode. At 13.22, the z-polarized Gaussian beam GHz normally comes from the upper-left corner. The dimension of a dielectric grating (black arrow) is 20 mm × 5 mm with the $\varepsilon = 9$. (a) The red arrow indicates that the MSP mode propagates along the x direction. The inset shows the average wavelength ($\bar{\lambda}_{MSP}$) of the MSP mode roughly is (106.467 – 90.707) /4 = 3.940 mm, which is almost identical the value of Eq. (2) ($\lambda_{MSP} = 3.947$ mm). (b) and (c) show that intensity of the $E_z(H_y)$ decays exponentially along the y direction at x = 100 mm.

4. Conclusion

We analytically derive the attenuation coefficients and dispersion equation of the MSP mode supported by an anisotropic magnetic medium, in which one requires to meet the criterion of ($\mu_x < 0 < \varepsilon_z$ and $\mu_x \mu_y > 1$) and momentum conservation by diffraction gratings. The designed PBMM inherently possesses biaxial magnetic responses accompanied negative permeability ($\mu_x = -4.20$, $\mu_y = -0.252$ at 13.22 GHz); meanwhile, the electric permittivity (ε_z) is retrieved as 3.46. Also, we numerically proved that the MSP mode can exist under the conditions ($\mu_x < 0 < \varepsilon_z$ and $\mu_x \mu_y > 1$) and momentum conservation. The average wavelength ($\overline{\lambda}_{MSP}$) of the MSP mode is calculated as 3.940 mm which is nearly correspondent to the analytical wavelength ($\lambda_{MSP} = 3.947$ mm). It ensures significantly the accuracy of numerical simulation.

References

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