

# An Update of Progress in Non-Foster Metamaterials

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## Abstract

One of the most recent research topics in metamaterial field deals with a possible use of non-Foster elements, which introduces a variety of novel properties such as the ultra-broad-band operation and superluminal effects. Here, the underlying physics and possible applications are reviewed. Several state-of-the-art examples of extremely broadband 1D and 2D active ENZ and MNZ metamaterials developed at University of Zagreb are presented and some new research directions are highlighted.

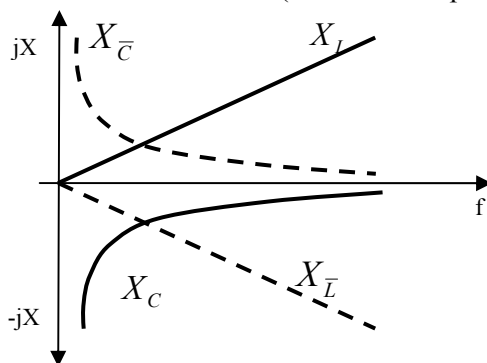
## 1. Introduction and state of the art

It is well known that every passive metamaterial must satisfy the basic dispersion constraints [1]:

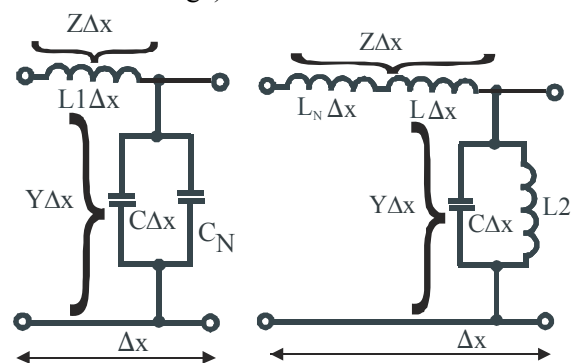
$$\partial[\omega \cdot \varepsilon(\omega)]/\partial\omega > \varepsilon_0, \quad \partial[\omega \cdot \mu(\omega)]/\partial\omega > \mu_0. \quad (1)$$

Here  $\varepsilon$  and  $\mu$  stand for the permittivity and permeability, respectively. The symbol  $\omega$  is angular frequency. The constraints given in (1) cause inherent narrowband behavior of all passive metamaterials (**Single-Negative**, **Double-Negative**, **Single-Near-Zero**, **Double-Near-Zero**). Equation (1) is actually an equivalent of the Foster reactance theorem in circuit theory [2] that requires  $(\partial X/\partial\omega) > 0$  and  $(\partial B/\partial\omega) > 0$  ( $X$  and  $B$  being reactance and susceptance, respectively). On the other hand, active electronic circuits that violate Foster theorem were introduced long time ago [3]. These circuits behave as negative capacitors or negative inductors that have dispersion curves that are the exact inverse of the dispersion curves of ordinary 'positive' elements (Fig. 1). Therefore, one could expect that the dispersion of ordinary passive metamaterials might somehow be combined with the 'inverse' dispersion of non-Foster elements, resulting in the broadband behavior.

Broadband ENG and MNG metamaterials based on an array of small dipoles loaded with negative capacitors or an array of small loops loaded with negative inductors were introduced theoretically in [4]. However, subsequent theoretical study [5] revealed that these 'purely negative' non-Foster metamaterials would be inherently unstable. This is in an agreement with known fact that the stability is assured if the net capacitance or the net inductance around a closed loop in an electric circuit is positive [6]. Moreover, our recent study [7] showed that the stability criteria can be much more complicated, depending on the topology of the circuit. Thus, broadband stable non-Foster SNG metamaterials do not seem to be feasible (at least at the present state of the knowledge).



**Fig. 1:** Reactance of positive (solid line) and negative non-Foster (dashed line) reactive elements

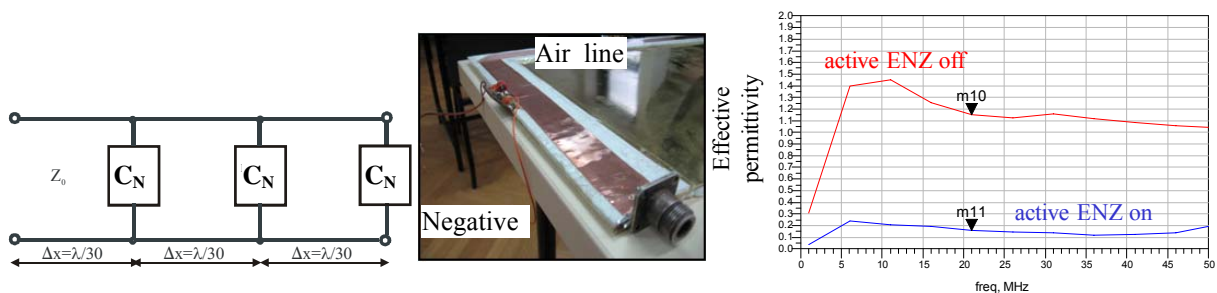


**Fig. 2:** Equivalent circuits of different non-Foster metamaterials, **Left:** 1D MNZ metamaterial, **Right:** 1D ENZ metamaterial

However, one may think of a transmission line periodically loaded with either series lumped negative inductors (Fig. 2 left) or shunt negative capacitors (Fig. 2 right). If the values of non-Foster elements are chosen in a way that assures positive values of inductance or capacitance one will get a stable structure. The equivalent permittivity and permeability of these transmission lines (or 1D metamaterials) are given by:

$$\varepsilon_r(\omega) = [C / \varepsilon_0 - |C_N| / (\varepsilon_0 \Delta x)], \quad \mu_r(\omega) = [L / \mu_0 - |L_N| / (\mu_0 \Delta x)] \quad (2)$$

Here  $C$  and  $L$  stand for distributed line capacitance and inductance, respectively. The symbols  $C_N$  and  $L_N$  stand for loading (lumped) non-Foster negative capacitors and inductors. From (2) it is obvious that a proper choice of  $C_N$  in the left part of Fig. 2 (or  $L_N$  in the right part of Fig. 2) will yield ( $0 < \varepsilon_r < 1$ ) or ( $0 < \mu_r < 1$ ) (i.e. the entirely dispersionless ENZ or MNZ metamaterial). This basic idea was presented in [8,9,10] and further improved in [11], where the experimental broadband 1D ENZ non-Foster-element-based metamaterial was reported. It was based on an air transmission line loaded with three op-amp-based negative capacitors (left and middle parts of Fig. 3).



**Fig. 3:** (Taken from [8]) **Left :** Three-unit-cell RF active ENZ transmission line based on negative capacitors, **Middle:** Experimental prototype, **Right:** Measurement results

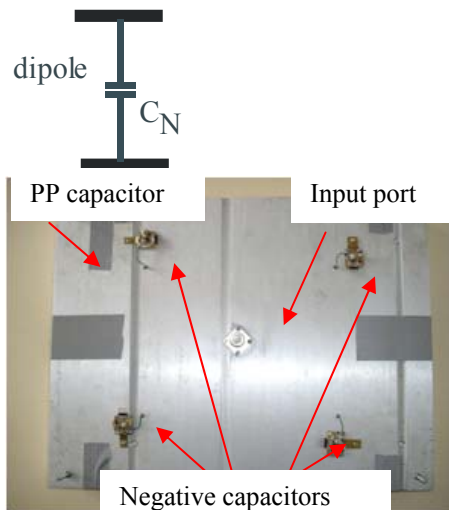
Measured real part of effective permittivity (the right part of Fig. 3) was rather constant (between 0.1 and 0.2), in the frequency range 10 MHz to 40 MHz (within two octaves). This bandwidth is significantly wider than a bandwidth of every passive ENZ metamaterial. On top of this, it was found that this type of transmission line supports superluminal both phase and group velocities [11]. Although these results look counter-intuitive, they are in a perfect agreement with causality [12].

### 3. New Research Directions

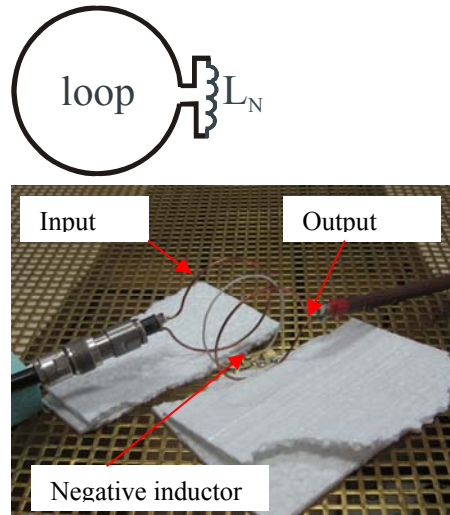
One of the issues that are currently being investigated is a possible use of non-Foster elements as the building blocks of metamaterial inclusions for 2D volume metamaterials. We followed the basic idea from [4], but only for the case of SNZ metamaterials (as explained before, the SNG non-Foster metamaterials do not seem to be feasible). The 2D broadband ENZ volume metamaterial based on RF negative capacitors developed in previous studies [10,11,12] was constructed and tested. It comprised four dipoles (copper wire with diameter of 1 mm and the length of 6 cm with top copper capacitive plates with dimensions of approximately 10 x 12 cm) (Fig. 4). Each dipole was centrally loaded with the negative capacitor operating in the range 2 MHz to 40 MHz. The four ‘inclusions’ were mounted into 60 cm x 40 cm x 6 cm parallel plate capacitor. If the ‘filling’ (an array of inclusions) operates as an continuous ENZ metamaterial, it should decrease the capacitance of the capacitor. Preliminary measurements of the input reflection coefficient of the capacitor (not shown here due to lack of space) revealed stable ENZ behavior within an octave bandwidth. This 2D active ENZ metamaterial could find application in 2D volumetric plasmonic cloaks.

In addition, the negative inductor based on a modified FET-based circuit from [9] was developed. It had inductance of -60 nH up to maximal operating frequency of 40 MHz. The prototyped negative inductor was used as a load of a simple small loop antenna (made out of 2mm diameter copper wire formed into 50 mm diameter loop) (Fig. 5). It is actually an active replica of a SRR. Additional positive capacitor was connected in a parallel with negative inductor in order to achieve MNZ behavior. This ‘active SRR’ was put between two loops that were connected to the ports 1 and 2 of a net-

work analyzer and the transmission coefficient ( $S_{21}$ ) was measured. Obtained results (not shown here due to lack of the space) showed a stable broadband operation that proved the basic idea. However, the unexpected strong influence of coupling loops on the behavior of the active inclusion was found. In some cases (for some values of mutual inductances between the inclusion and the loops), the MNZ behavior was completely suppressed. The possible remedy for this problem might be optimization of coupling coefficient, but this needs further investigation.



**Fig. 4: Upper :** A short dipole loaded with negative capacitor, **Lower :** Experimental realization of  $2 \times 2$  2D array



**Fig. 5: Upper :** A loop loaded with negative inductor, **Lower :** Experimental realization

#### 4. Conclusions

A concept of active non-Foster metamaterial was reviewed and several state-of-the-art practical examples were pointed out. Some new research directions and possible applications were highlighted. More numerical and experimental results will be presented at the conference.

#### Acknowledgment

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