Wave impedance retrieving via Bloch modes analysis

A. Andryieuski[1], S. Ha[2], A. Sukhorukov[2], R. Malureanu[1], Y. Kivshar[2] and, A.V. Lavrinenko[1]

[1]DTU Fotonik – Department of Photonics Engineering,
Technical University of Denmark, Bld. 343, DK-2800, Kgs. Lyngby, Denmark
Phone: +45 4525 6392; Fax: +45 4593 6581; email: alav@fotonik.dtu.dk
[2]Nonlinear Physics Centre, Research School of Physics and Engineering, Australian National University, Canberra, ACT 0200, Australia, Fax: + 358–94512152;

Abstract

The main bottleneck in the restoration of electromagnetic effective parameters is connected to the impedance retrieving. The S-parameters method gives the input (Bloch) impedance, which, being then used for permittivity and permeability determination, causes some fundamental physics principles violation, like antiresonance behaviour with Im(ε) < 0, Im(μ) < 0. We employ the Bloch mode analysis of periodic metamaterials to extract the dominating (fundamental) Bloch mode. Then it is possible to determine the Bloch and wave impedances by the surface and volume averaging of the electromagnetic field of the Bloch mode, respectively. Case studies prove that our approach can determine material and wave effective parameters of lossy and lossless metamaterials. In some examples when the passivity is violated we made further analysis and showed that this is due to the failure of concept of impedance retrieving through the volume averaging.

1. Introduction

Artificially structured metamaterials (MMs) attract the attention of scientists and engineers due to their unprecedented electromagnetic properties. Negative refractive index, very large or near zero permittivity and permeability, giant optical activity - these are just few examples of what MMs can provide [1]. It is convenient to describe the properties of the MMs with effective parameters (EPs), such as refractive index n, impedance z, permittivity ε and permeability μ, provided that these EPs can be introduced [2]. The EPs simplify significantly description of the MMs behaviour in terms of observable phenomena such as the propagation of electromagnetic waves inside a MM slab and their reflection and transmission at the MM slab interfaces. The simplest way to restore EPs is to assign reflection and transmission coefficients calculated or measured for a MM slab to a slab of the same thickness made from a uniform material. However, it often leads to violation of some fundamental physics principles [2]. The extensive discussion in literature reflects the fact that there is no universal rule to introduce EPs in general case of three-dimensional metal-dielectric composites with complex topology. The state-of-the-art conclusion is that retrieved EPs are of two types: [2-4]: *material* (or *local*) effective parameters ε_M and μ_M and *wave* (or *non-local*) effective parameters ε_W and μ_W. The latter are usually restored from the reflection and transmission coefficients for the MM slab [5] and they may allow one to calculate the reflection and transmission of the MM slab of another thickness. It is obvious that the reflection from a MM slab should depend on whether the MM slab termination coincides with the border or with another cross-section in the middle of the unit cell, so the input impedance and the wave EPs depend on the MM opening cross-section. Material EPs depend only on the properties of the metamaterial. In a general case wave and material parameters are not equal.

The discrepancy between retrieved wave ε_W and μ_W and material ε_M and μ_M parameters as it was underlined in [6] is connected with the inadequate relevance between surface-averaged and volume-averaged fields, which further proceeds to surface impedances or wave impedances correspondingly. Therefore we focus in this paper on the impedance retrieving analysing wave propagation inside bulk metamaterial.

2. Impedance retrieving: theory

Accordingly to [6] the bulk wave impedance is defined through the ratio of fundamental (dominating) Bloch mode volume-averaged linearly-polarized electric and magnetic fields

$$Z_W = \frac{E_x^V}{z_0 H_y^V}, \tag{1}$$

while Bloch impedance is derived from the surface averaged fields

$$Z_B = \frac{E_x^S}{z_0 H_y^S}, \tag{2}$$

where z_0 is the free-space impedance. We excite the MM slab, which consists of the periodically arranged unit cells with a linearly-polarized plane wave propagating along the z-axis. So the simulated fields of the waves propagating inside a MM slab should first undergo the Bloch modes expansion. The slab should be thicker than 3-4 MM monolayers for that we can neglect the so-called Drude transition layers [2]. Full/wave simulations are done by the CST Microwave Studio [7] in time-domain. We use the high-resolution spectral analysis method [8] to decompose the total field into a sum of Bloch modes. The only information required for the application of this method is the knowledge about the number of strongest Bloch modes M excited in the structure. Then we extract the wavenumbers K_m and field profiles E_m, H_m of all forward and backward propagating Bloch modes at each frequency. By monitoring the accuracy of such decomposition, we also check whether other Bloch modes may have significant excitation amplitudes, and if this is a case we can increase the number M and repeat the extraction procedure. After extraction of fundamental Bloch modes we perform either surface or volume field averaging leading us to the surface (Bloch) Z_B (2) or wave Z_W (1) impedances accordingly.

3. Impedance retrieving: numerical examples

We tested the method on several examples such as a homogeneous slab with Lorentz dispersion in both ε and μ; a set of the nanospheres with the plasmonic resonances; split cubes MM with negative permeability; wire medium with negative permittivity; fishnet MM and split cube in carcass MM. In all cases, the MM slab consisted of 10 monolayers. We compared results with the EPs restored by the S-parameters approach for three monolayers thick slab. In this submission we present results for the set of silver nanospheres. Metallic nanospheres of radius 30 nm being arranged in the regular structure will provide a MM with permeability close to 1, since the nanospheres are non-magnetic. Silver is considered as the Drude metal. Results are shown in Fig.1. Effective refractive indices restored with the different approaches are identical (see Fig.1a). Bloch impedance Z_B, retrieved with the surface averaging is the same as the one restored with the S-parameters method. The wave impedance Z_W (see Fig.1b, circles) differs from Bloch impedance, which shows several resonances.

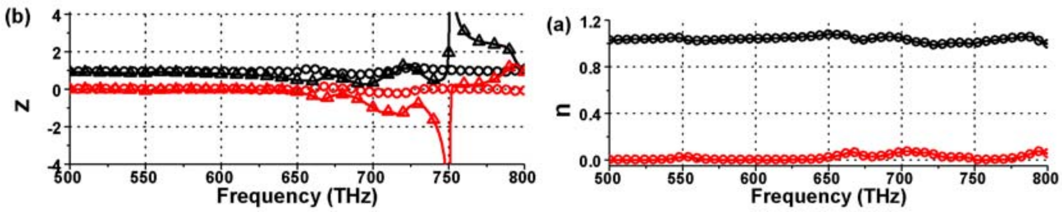

Fig. 1: Effective parameters of the MM consisting of plasmonic nanospheres: refractive index (a), impedance (b), real (black) and imaginary (red) parts. Results by volume-averaged (circles) and surface-averaged fields (triangles) are compared with the S-parameters method (solid lines)

The electromagnetic EP (permittivity ε and permeability μ) restored from the impedance –refractive index spectra are presented in Fig.2. The magnetic permeability μ_B shows non-physical negative imaginary part (Fig.2b), so-called antiresonance that normally would correspond to the gain in the system. However, material EPs, restored through the wave impedance Z_W, do not exhibit any antiresonant behaviour. The small negative values of permittivity are assigned to the calculation errors.

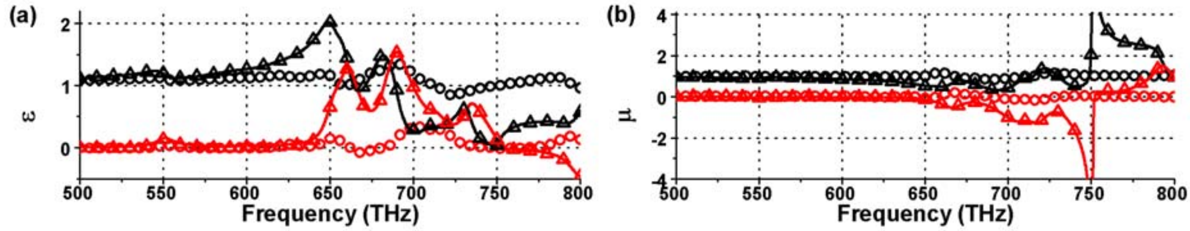

Fig. 2: Effective parameters of the MM consisting of plasmonic nanospheres: permittivity (a) and permeability (b), real (black) and imaginary (red) parts. Results by volume-averaged (circles) and surface-averaged fields (triangles) are compared with the S-parameters method (solid lines)

4. Conclusion

Our field averaged Bloch mode restoration method is able to retrieve both material and wave EPs for a wide range of materials, which can be lossy or lossless. Both restoration (wave and material EPs) are performed within a single computational cycle, because fields on the unit cells entrance facets or in its volumes are available and can be exported from Maxwell's solver. The approach has no limitations on a MM slab thickness. Moreover, restored material EPs have no non-physical "anti-resonances" or spurious magnetic behaviour. Concerning the method constraints, we must admit that in some cases volume-averaged fields approach retrieves negative real part of the impedance [9]. The closer look on the phenomenon shows that in such regions the component of the Poynting vector in the direction of propagation of the Bloch mode differs significantly from that of the raw fields. It manifests that the wave impedance cannot be introduced in such examples mostly due to pronounced spatial dispersion properties of materials. Nevertheless, the refractive index (wavenumber) dispersion can be restored accurately even in such examples. It is very important to stress that the Bloch mode analysis can be applied to restore parameters from the measured data, see, for example [10].

References

[1] Metamaterials Handbook, F. Capolino, ed., CRC Press, 2009.
[2] C. Simovski, Material parameters of metamaterials (a Review), *Optics Spectroscopy,* vol. 107, 726-753, 2009.
[3] C. Menzel, C. Rockstuhl, T. Paul, F. Lederer, and T. Pertsch, Retrieving effective parameters for metamaterials at oblique incidence, *Physical Review B*, vol.77, 195328, 2008.
[4] C.R. Simovski, Bloch material parameters of magneto-dielectric metamaterials and the concept of Bloch lattices, *Metamaterials*, vol.1, 62-80, 2007.
[5] D.R. Smith, S. Schultz, P. Markos, and C.M. Soukoulis, Determination of effective permittivity and permeability of metamaterials from reflection and transmission coefficients, *Physical Review B* vol. 65, 195104, 2002.
[6] C.R. Simovski, On electromagnetic characterization and homogenization of nanostructured metamaterials, *Journal of Optics A*, vol.13, 013001, 2011.
[7] CST Computer Simulation Technology AG, http://cst.com.
[8] S. Ha, A. A. Sukhorukov, K. B. Dossou, L. C. Botten, C. M. de Sterke, and Y. S. Kivshar, Bloch-mode extraction from near-field data in periodic waveguides, *Optics Letters*, vol.34, 3776-3779, 2009.
[9] A. Andryieuski, S. Ha, A. A. Sukhorukov, Y. S. Kivshar, and A. V. Lavrinenko, Bloch-mode analysis for unambiguous retrieval of metamaterial effective parameters, arXiv:1011.2669v1.
[10] S. Ha, Andrey A. Sukhorukov, A. V. Lavrinenko, I. V. Shadrivov, D. A. Powell, and Y. S. Kivshar, Observation of tunneling of slow and fast light in coupled periodic waveguides", *Applied Physics Letters*, vol.98, 061909, 2011.