Tunable Nonlinear Response of Coupled Resonators

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Abstract

We investigate the effect of tuning various linear properties of two coupled split-ring resonators (SRRs) via altering their relative positions. We also study the nonlinear properties of the coupled SRRs and the tunability of their collective nonlinear response.

1. Introduction

In 1999, Pendry et al. [1] proposed the concept of the Split-Ring Resonator (SRR), a building block of composite metamaterials which can exhibit a negative magnetic response due to its strongly resonant magnetic polarizability. Strong interactions occur between the rings in an array due to the complex near-field patterns in the metamaterial.

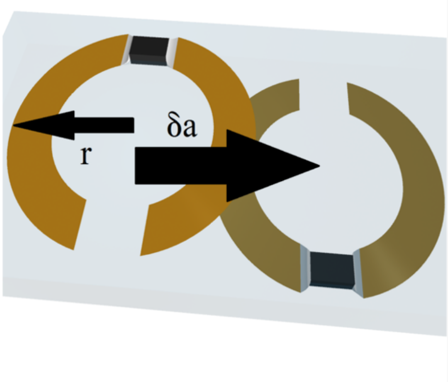
The tunability of such structures has been studied extensively, based on the control of the relative arrangement of the SRR's [2, 3], showing the ability to tune the properties of the metamaterial. These properties can also be tuned externally by inserting inclusions such as diodes into the rings [4, 5], allowing tuning in the system due to the changing of the external fields. While the nonlinear tunability of an individual ring (or edge-coupled pair) has been studied in depth, the links between tuning the linear properties of the system and its nonlinear response has not been addressed before.

Here we study both the linear tunability of various properties of a resonant system due to the relative shifting of two broadside-coupled SRR's, and the effect this shifting has on the nonlinear response due to diodes inserted into the system.

2. Tunability of Linear Properties

For our experiments we use two copper rings which are 3.75mm in outer radius, 3.25mm in inner radius, and have a gap width of 1mm. These rings are printed back to back on 1.6mm thick FR4 circuit board, at various values of δa, as seen in Fig. 1(a). Each ring has a second gap of 0.4mm opposite the main gap, across which a Skywork's SMV1405-079 series diode is attached. The rings are mounted in a plexiglass stand with a dielectric of about 2.2, so as to improve the mechanical stability of the experiment, ensuring the reproducibility of the results. The incoming microwaves are polarized so that the H field is perpendicular to the loops.

Experimental results were measured using a Rohde and Schwarz network analyzer in a rectangular waveguide. The most meaningful property to measure is the energy absorption, which describes the frequency of maximum current distribution on the rings, and is given by $1 - |S_{21}|^2 - |S_{11}|^2$, where S_{21}



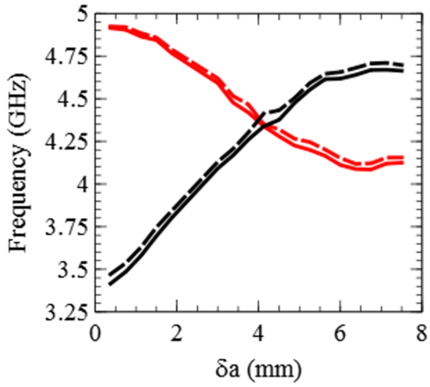


Fig. 1: (a) Schematic of the shifting SRRs with inserted diodes. δa is the relative shift between the two rings. (b) Resonant frequencies of both symmetric (black) and antisymmetric (red) resonances at both -20dBm (solid) and 15dBm (dashed) power inputs, over the range of relative shifts (δa).

and S_{11} are the transmission and reflection coefficients respectively. The absorption curves are measured experimentally for values of δa between 0 and 7.5mm, in 0.375mm increments. The value of δa is defined as the distance between the centers of the rings.

The maximums of the resonant peaks are found and plotted as a function of δa in Fig. 1(b). In this system, two resonant modes are present - symmetric and antisymmetric - which are tunable through the changing of δa [2, 3]. These results are qualitatively similar to those found in Ref. [3], however, due to the addition of the plexiglass stand, the antisymmetric mode is much more strongly excited. This allows us to experimentally view both modes for all values of δa. The resonant frequencies are determined by the combined capacitance of the diode in series with the SRR. As consistent with the findings in Ref. [3], there are strong responses in both modes due to the changing of δa.

Other features of the absorption curve that we are interested in tuning are the width of the resonance, which is directly related to the quality factor Q, and the maximum absorptions. The width is determined by the losses in the system, primarily due to radiation. By shifting the rings, we are altering the electric and magnetic dipole positions, which determine the radiation losses occurring in both modes. This enables us to tune the width significantly, as shown in Fig. 2(a). Related to this is the height (or maximum absorptions) of the resonances, which we can also tune significantly, as shown in Fig. 2(b). In both cases, increasing δa causes a increase (decrease) in the value for the symmetric (antisymmetric) mode, though the height of the antisymmetric mode also increases initially. Being able to tune these properties is important as it shows we can have control both the resonant frequencies and the quality.

3. Tunability of Nonlinearity

This tunability of the linear properties leads us to the investigation of the tunability of the nonlinear response of the system. Due to the inserted diodes in the system, by altering the input power, and hence the external fields, we can also tune the resonances nonlinearly. By changing the intensity of the incoming waves, we are changing the effective capacitance of the system, shifting the resonant frequencies.

Again, we have measured the absorption curves for the changing value δa, this time at both input powers -20dBm and 15dBm. The resulting difference between the two resonant frequencies for each value of δa is shown in Fig. 2(c). As can be seen, there is a significant effect here. While the relationship between the relative shift and the nonlinearity cannot necessarily be quantified, it is clear that the nonlinearity is much stronger for the symmetric resonance (and weaker for the antisymmetric) when the rings are closer together. Also worth noting is that in Fig. 1(b), the resonant frequencies at higher values of δa begin to

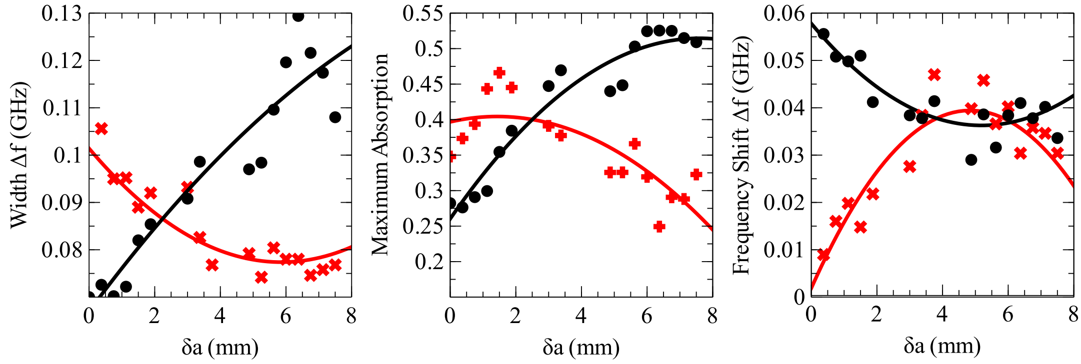


Fig. 2: (a) Experimental 3dB widths of the resonances as a function of relative shift (δa) for both symmetric (circles) and antisymmetric (crosses) modes. (b) Heights of both the symmetric (circles) and antisymmetric (crosses) resonances, over the range of relative shifts (δa). (c) Nonlinear shift in resonant frequencies for symmetric (circles) and antisymmetric (crosses) for relative shifts (δa). Solid lines are the second order polynomial fits to the data.

converge slightly where the nonlinearity starts to re-diverge.

By comparing Figs. 2(a-c), we can see that there is a link between the changing linear properties and the nonlinearity in the system. Initially, the width of the resonance appears to be inversely related to the nonlinearity, with the values crossing at almost the same value of δa. This crossing of values also occurs with the heights in Fig. 2(b).

4. Conclusion

We have shown that by shifting two coupled split ring resonators relative to each other, we can significantly control the properties of both the symmetric and antisymmetric resonant modes in the system. This includes controlling the Q factor and the nonlinear response. Further work remains in optimizing the tunability of the nonlinear properties in this system, and in investigating similar effects in other systems.

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