# Topology optimized acoustic and all-dielectric optical cloaks

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**Abstract**

In this paper we present an alternative way of designing electromagnetic and acoustic cloaks based on isotropic and non-extreme materials. Using the standard gradient-based topology optimization method we vary the isotropic material properties within the cloak in an iterative approach, such that the norm of the scattered (electric, magnetic or acoustic) field in the surroundings is minimized. Both optimized graded-index designs and optimized designs based on circular inclusions in a background material are presented. For the specified angle of incidence the cloaking properties is shown numerically to be nearly perfect in a limited frequency range.

## 1. Introduction

In order to hide a given object for a specific frequency range in the acoustic or electromagnetic spectrum it is necessary to obtain the material properties of these so-called acoustic or electromagnetic cloaks and be able to realize the properties. The material properties, for electromagnetic or acoustic cloaks can be derived by a coordinate transformation in transformation optics [1] or transformation acoustics [2], respectively. Several simplified electromagnetic cloaks have been realized (e.g. [3]) and all-dielectric carpet cloaking has been demonstrated [4]. In the acoustic case realizations of the material properties derived by transformation acoustics are more cumbersome due to the anisotropic mass density, which is not common in naturally occurring materials. A recent realization, in which they make use of an transmission line approach to overcome the above challenge, of an acoustic cloak in water waves for ultrasound has been demonstrated [5]. All the theoretical work and realizations of electromagnetic and acoustic cloaks are to the authors knowledge based on anisotropic material parameters, however, we have in this work systematically addressed the intriguing question: How efficiently can we cloak when using conventional simple isotropic media readily available in nature?. Results [6] show that gradient-based topology optimization [7] can be used to find the permittivity distribution for a low-contrast all-dielectric optical cloak that hides a perfectly electric conducting cylinder in a limited frequency range for up to 4 symmetrical distributed angles of incidence. The physics of time-harmonic acoustic waves and time-harmonic $E_z$- or $H_z$-polarized electromagnetic waves are governed by an identical mathematical form; the scalar Helmholtz equation as shown below in equation (1). The differences in the Helmholtz equation for acoustic, $E_z$- and $H_z$-polarized waves are found in the material properties and state variable; mathematical the equations are the same form. Thus with a limited reformulation, the initial methodology of designing optical cloaks can also be used to design an acoustic cloak with isotropic material properties to circumvent the problems of the anisotropic mass density.

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| Wave-type | $u$ | $A$ | $B$ |
|---|---|---|---|
| $E_z$-polarized | $E_z$ | $\mu_r^{-1}$ | $\epsilon_r$ |
| $H_z$-polarized | $H_z$ | $\epsilon_r^{-1}$ | $\mu_r$ |
| Acoustic | $p$ | $\rho_r^{-1}$ | $\kappa_r^{-1}$ |

Tab. 1: Parameter relations for a general notation of Helmholtz' equation.

## 2. Method

The scalar Helmholtz equation governs the physics of both an $E_z$-polarized, $H_z$-polarized and acoustic wave and in a general form we can state this as

$$\nabla \cdot \big(A\nabla u\big) + k_0^2 B u = 0 \tag{1}$$

where $A$, $B$ and $u$ are given in Tab. 1, $k_0 = \omega/c$ is the free space wave vector, $\omega$ is the angular frequency, $c = (\epsilon_0\mu_0)^{-1/2}$ is the speed of light in vacuum for an electromagnetic wave or $c = (\rho_0\kappa_0^{-1})^{-1/2}$ is the speed of sound in air for an acoustic wave, $\epsilon$ is the permittivity, $\mu$ is the permeability, $\rho$ is the density, $\kappa$ is the bulk modulus, the subscripts $r$ and $0$ denotes the relative parameters to that of free space and air for the electromagnetic and acoustic wave, respectively.
The problem is modeled in 2D using 3 concentric circles and solved using the finite element method [8]. The inner, middle and outer domain represent the hidden cylinder, the cloak and the surroundings, respectively. The material of the hidden cylinder is iron for the acoustic problem and a perfectly electric conductor for the electromagnetic problem. Using the standard gradient-based topology optimization method [7] we vary the isotropic material properties $A$ and $B$ within the cloak in an iterative approach, such that the norm of the scattered (electric, magnetic or acoustic) field in the surroundings is minimized. In case of the electromagnetic cloak we set the relative permeability to 1 in order to mimic a non-magnetic material. Hence for the electromagnetic problem the relative permittivity, $\epsilon_r$, is varied continuously on an element basis between two materials using a linear interpolation and for the acoustic problem the relative density, $\rho_r$, and the bulk modulus, $\kappa_r$, are varied. Without any penalization of intermediate material properties a graded index material is obtained from the optimization. Hence the Material-Mask Overlay Strategy (MMOS) [9] is be used in order to achieve designs without intermediate material properties. Furthermore the shape of one material is confined to circles, which make the designs easier to realize. The MMOS formulation results in an optimization problem, in which the position and radii of the prescribed circles are varied.

## 3. Results

Using the method outlined we have optimized for an $E_z$-polarized and an acoustic wave. In Fig. 1 (a) and (f) the non-cloaked cylinder illuminated by a uniform plane $E_z$-polarized and acoustic wave are presented, respectively. The interference pattern from the scattered field is very notable, especially as a shadow region behind the cylinders. For the $E_z$-polarized problem we interpolate the relative permittivity, $\epsilon_r$, between 1 and 6 for the graded design in Fig. 1 (b). The design obtained from MMOS results in circular inclusions with a relative permittivity of 3 in a background material of a relative permittivity of 1 in Fig. 1 (d). The circular inclusions gives a layout of effective permittivity that mimics the graded design. Both optimized designs for the $E_z$-polarized wave are basically waveguides which guide and delay the waves inside the cloak and phase match them to the waves outside. For the acoustic problem we interpolate the relative density, $\rho_r$, between 1 (air) and $6.54 \cdot 10^3$ (iron) and the relative bulk modulus, $\kappa_r$, between 1 (air) and $1.20 \cdot 10^6$ (iron). The graded design with intermediate material properties and resulting field is presented in Fig. 1 (g) and (h), respectively. A physical interpretation of the graded

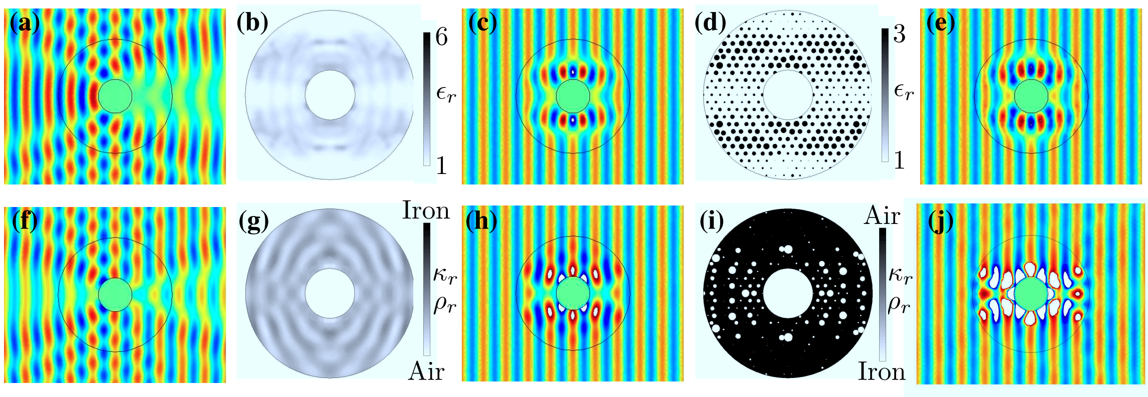


Fig. 1: The cloak designs and FE simulations for an $E_z$ polarized wave (a)-(e) and an acoustic wave (f)-(j). (a) and (f) show the non-cloaked cylinders. (b), (c), (g) and (h) show the graded designs and resulting fields. (d), (e), (i) and (j) show the designs obtained using MMOS and resulting fields.

design for the acoustic problem with a linear interpolation between air and iron is not straight-forward, but included here as a reference. Using MMOS a design containing iron-rods of varying radii and position are obtained and presented in Fig. 1 (i) with the resulting field shown in Fig. 1 (j). However, the presented designs for both the electromagnetic and the acoustic problem are both highly resonant to the frequency and highly localized to the considered angle of incidence. Hence a perturbation in either the frequency or angle of incidence have a negative effect on the cloaking properties of the designs.

## 4. Conclusion

We have numerically demonstrated that topology optimization can be used to find the layout of isotropic material in both electromagnetic and acoustic cloaks. Both optimized graded-index designs and optimized designs based on circular inclusions in a background material are presented. For the specified angle of incidence the cloaking properties is nearly perfect in a limited frequency range. The method of designing cloaks can further be extended in several ways, e.g. by optimizing for wider frequency ranges, radar cross sections or directive properties.

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