Seismic cloaks

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Abstract

We analyse cylindrical cloaks designed to control bending waves and coupled in-plane pressure and shear waves in elastic plates. Whereas electromagnetic cloaks require symmetric rank 2 tensors of permittivity and permeability, our approach is based upon the introduction of an asymmetric rank 4 tensor of elasticity and a scalar density. For the special case of bending waves in thin plates, the cloak is described by a rank-2 Young tensor. Applications are in passive anti-earthquake systems.

Introduction:

In 2006, Pendry et al. [1] and Leonhardt [2] independently showed the possibility of designing a cloak that renders any object inside it invisible to electromagnetic radiation. The experimental validation [3] of these theoretical considerations was given, a few months later, by an international team involving the former authors who used a cylindrical cloak consisting of concentric arrays of split ring resonators. This cloak makes a copper cylinder invisible to an incident plane wave at 8.5 GHz as predicted by the numerical simulations.

A natural question is to check whether or not such cloaking applies to other types of waves. It turns out that the answer is positive for acoustic waves [4,5,6,7,8], and this has been experimentally validated for surface water waves with a micro-structured metallic cloak at 10 Hz [9].

However, when one moves to the area of elastic waves, there is no straightforward one-to-one correspondence between Navier's and acoustic wave equations, and useful analogies with the theory of electromagnetic cloaking seem to break down [10]. In this talk, we will explain why in the special case of thin elastic plates, one can still implement the geometric transform of Pendry et al. [1]. This unexpected positive outcome comes from structural similarities between the harmonic and biharmonic equations, which were recently exploited in the analysis of stop band for periodic thin plates [11], see Fig. 1(a).

Let us briefly discuss the elastic properties of the ideal cloak for coupled in-plane pressure and shear waves, see Fig. 1(a) and [13] and then focus on being waves in thin plates, see Fig. 1(b) and [12].

Transformation elastodynamics:

Let us consider an in-plane displacement field **u**=(u1,u2,0) solution of the Navier equation

$$\nabla \cdot \mathbb{C} : \nabla \mathbf{u} + \rho \omega^2 \mathbf{u} = \mathbf{0} \ ,$$

Where C is the tensor of elasticity, rho is the density and omega is the wave frequency.

Under the geometric transform of Pendry et al. [1]

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$$\begin{cases} r' = r_0 + \frac{r_1 - r_0}{r_1} r \ , \ \theta' = \theta \ , & \text{for } r \leq r1 \\ r' = r \ , \ \theta' = \theta \ , & \text{for } r > r1 \end{cases}$$

this equation takes the following form [12]:

$$\nabla \cdot \mathbb{C}' : \nabla \mathbf{u} + \rho' \omega^2 \mathbf{u} = \mathbf{0} \ ,$$

Where the density is a spatially varying scalar function

$$\rho' = \frac{r - r_0}{r} \left(\frac{r_1}{r_1 - r_0} \right)^2 \rho$$

And the tensor of elasticity only retains the minor symmetries

$$\begin{array}{ll} \mathbb{C}'_{rrrr} = \frac{r-r_0}{r}(\lambda + 2\mu), & \mathbb{C}'_{\theta\theta\theta\theta} = \frac{r}{r-r_0}(\lambda + 2\mu), \\ \mathbb{C}'_{rr\theta\theta} = \mathbb{C}'_{\theta\theta rr} = \lambda, & \mathbb{C}'_{r\theta\theta r} = \mathbb{C}'_{\theta rr\theta} = \mu, \\ \mathbb{C}'_{r\theta r\theta} = \frac{r-r_0}{r}\mu, & \mathbb{C}'_{\theta r\theta r} = \frac{r}{r-r_0}\mu, \end{array}$$

One should note that on t he inner boundary r0 of the cloak, some coefficients vanish and other diverge. The physical interpretation for this is that infinite anisotropy is required in order to compensate for the longer trajectories followed by e lastic waves that are detoured around the invisibility region. The same kind of infinite anisotropy occurs in the permittivity and permeability tensors in electromagnetic invisibility cloaks [1].

Simplified parameters for thin plates:

In the case of thin plates, the problem becomes more tractable, as **u**=(0,0,U) is an out-of-plane displacement governed by a scalar equation:

$$< \lambda > \nabla.(\underline{\underline{\zeta}}^{-1} \nabla (< \lambda > \nabla.(\underline{\underline{\zeta}}^{-1} \nabla U))) - \beta_0^4 \, U = 0 \ ,$$

Where beta0 is a spectral parameter encompassing the rigidity of the plate, and

$$< \lambda >= \int_0^1 \rho^{-1/2}(r) \, dr$$

and

$$\underline{\underline{\zeta}} = \mathrm{Diag}(< \zeta^{-1} >^{-1}, < \zeta >) = \underline{\underline{E}}^{-1/2} \ .$$

Such a simple set of parameters allows for a simple structured design of the cloak, see Fig. 1(b).

Conclusion:

In this paper, we reviewed some design of elastodynamic cloaks. We note that recent experiments on ultrasonic cloaks [14] making use of locally resonant structures are also encompassed by the approach of transformed Navier equations, but in that case the elasticity tensor reduces to a 2 by 2 matrix.

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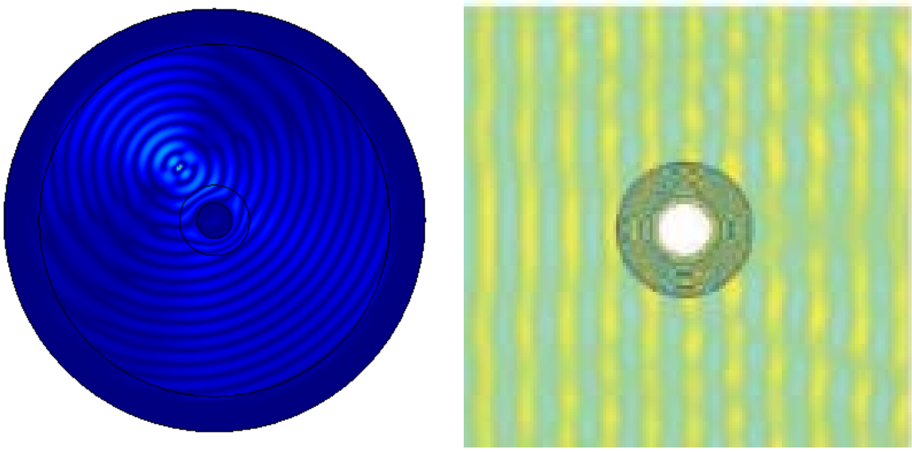

(a) ***(b)***

Fig. 1 (a) Magnitude of the in-plane displacement field for an elastic cloak achieving the control of coupled pressure and shear waves in presence of a point force.(b) Out-of-plane displacement field for a multi-layered elastic cloak achieving the control of bending waves over a large bandwidth.

