Zero-backscattering self-dual object from two chiral particles

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Abstract

In this paper we study two canonical chiral particles as a possible element for composite media. We write the polarisation components for one chiral particle, and see that when combined with the second, we need only four of the ten components to describe the scattered field with good accuracy. For the normal incidence, the unit element scatters as a Huygens' source with in-plane isotropy for linear polarisation. The reason behind this zero-backscattering behavior is that the element is self dual with 90-degree rotational symmetry. The scattered fields from a circularly polarised field are studied in detail, and it is seen that the scattered field has also the Huygens' pattern and the polarisation is circular.

1. Introduction

An element formed by two chiral particles can form an object that can radiate circularly polarised waves with a Huygens' pattern when the particles are fed with the proper phase shift [1]. It has been very recently realized that when illuminated with a plane wave, the element only forward scatters as a Huygens' source with very little backscattering [2]. For a realization of this effect, the incident plane wave needs to be coupled to one particle via e.g. the electric field and to the other via the magnetic field. Because the chiral particle has both electric and magnetic responses for both fields, a set of scattering sources is created that radiates as a Huygens' source if the chiral particle has the right dimensions. In the paper we write the polarisability components for the particles in a simple case and see that they form a certain self-dual object in the sense of electric and magnetic response, which is the cause of the observed scattering phenomenon. The use of the element as a unit cell of a composite material is discussed.

2. Chiral element of two canonical chiral particles

Two chiral particles oriented along x and y directions, named A and B, respectively, are presented in Fig. 1(a). The length of the dipole arms is l and the radius of the loop is a (the loop area $S = \pi a^2$). To study the scattering from the particles, we write the electric and magnetic polarisabilities of the small particles with the help of dyadic algebra:

$$\mathbf{p} = (\overline{\overline{a}}_{\rm ee} + \overline{\overline{b}}_{\rm ee}) \cdot \mathbf{E} + (\overline{\overline{a}}_{\rm em} + \overline{\overline{b}}_{\rm em}) \cdot \mathbf{H}, \qquad \mathbf{m} = (\overline{\overline{a}}_{\rm me} + \overline{\overline{b}}_{\rm me}) \cdot \mathbf{E} + (\overline{\overline{a}}_{\rm mm} + \overline{\overline{b}}_{\rm mm}) \cdot \mathbf{H}, \qquad (1)$$

where $\overline{\overline{a}}$ and $\overline{\overline{b}}$ refer to the polarisabilities of particles A and B, respectively. Each particle can possibly have ten scattering components [3], but reliable results can be obtained with only four components as was done in [2]. If we neglect the electric polarisability components for the loops and the mutual coupling

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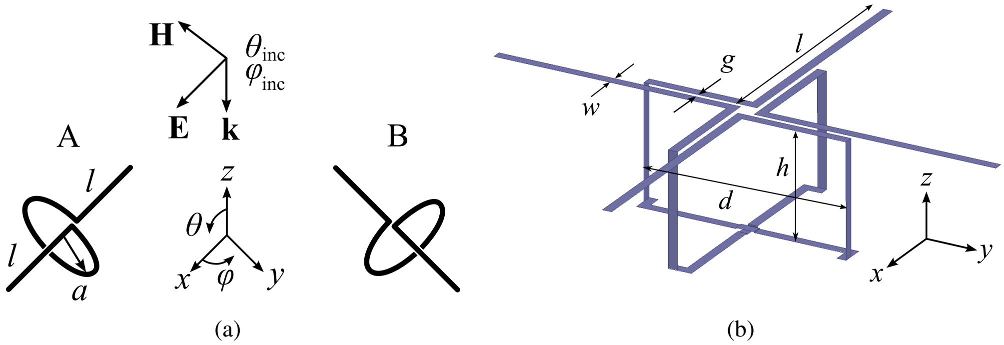


Fig. 1: a) Chiral unit element is composed of two chiral particles. (b) The simulation model: l = 27.0 mm, $d = 22.5$ mm, $h = 13.0$ mm, $g = 0.5$ mm, and $w = 1.0$ mm.

between the particles, the dipole moments (1) reduce to

$$\mathbf{p} = (a_{\mathrm{ee}}^{(xx)}\mathbf{x}_0\mathbf{x}_0 + b_{\mathrm{ee}}^{(yy)}\mathbf{y}_0\mathbf{y}_0)\cdot\mathbf{E} + (a_{\mathrm{em}}^{(xx)}\mathbf{x}_0\mathbf{x}_0 + b_{\mathrm{em}}^{(yy)}\mathbf{y}_0\mathbf{y}_0)\cdot\mathbf{H}, \tag{2}$$

$$\mathbf{m} = (a_{\mathrm{me}}^{(xx)}\mathbf{x}_0\mathbf{x}_0 + b_{\mathrm{me}}^{(yy)}\mathbf{y}_0\mathbf{y}_0)\cdot\mathbf{E} + (a_{\mathrm{mm}}^{(xx)}\mathbf{x}_0\mathbf{x}_0 + b_{\mathrm{mm}}^{(yy)}\mathbf{y}_0\mathbf{y}_0)\cdot\mathbf{H}, \tag{3}$$

where $\mathbf{x}_0$ and $\mathbf{y}_0$ are the x and y-directed unit vectors, respectively. $a_{\mathrm{ee}}^{(xx)}\mathbf{x}_0\mathbf{x}_0$ (reading from right to left) means that an x-directed electric field causes an x-directed electric dipole moment, etc. It can be calculated that the dipole arm and the loop area are in balance when $Sk = l$, where $k = \omega\sqrt{\mu\epsilon}$, i.e. the dipole and loop radiate equal magnitudes in the far zone [1, 2]. If the dipoles are at the origin, the scattering pattern for the normal incidence (from either $\pm z$ directions) is the Huygens' pattern with the directivity $D_{\mathrm{sca}} = \frac{3}{4}(\cos\theta \mp 1)^2$ and the polarisation is circular in all directions, right-hand circular polarisation (RHCP) for right-handed particles.

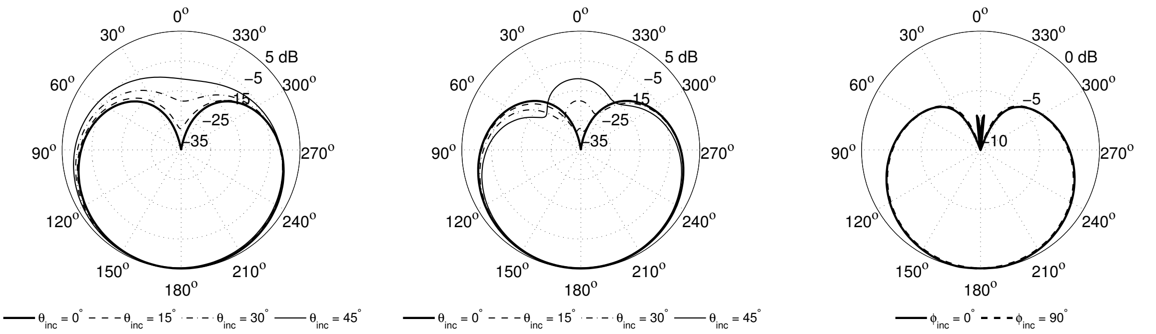
Is it a coincidence that the chiral element formed by two chiral particles does not seem to backscatter? No, the answer is given in [4]: for zero backscattering the object must have 90-degree rotational symmetry relative to the incident direction and self-dual characteristics. The dipole moments of the chiral element for the normal incidence (with $Sk = l$, and x-polarised $\mathbf{E}$) reduce to

$$\mathbf{p} = \frac{l^2}{j\omega}\frac{E}{Z_{\mathrm{tot}}}\mathbf{x}_0 + \eta\frac{l^2}{\omega}\frac{E}{Z_{\mathrm{tot}}}\mathbf{y}_0, \qquad \mathbf{m} = -\eta\frac{l^2}{\omega}\frac{E}{Z_{\mathrm{tot}}}\mathbf{x}_0 + \eta^2\frac{l^2}{j\omega}\frac{E}{Z_{\mathrm{tot}}}\mathbf{y}_0, \tag{4}$$

where $\eta = \sqrt{\mu/\epsilon}$ and Z_{tot} is the total impedance of the particle [3]. The polarisabilities (4) are self dual: $\mathbf{p}_{\mathrm{d}} = j/\eta\mathbf{m} = \mathbf{p}$ and $\mathbf{m}_{\mathrm{d}} = -j\eta\mathbf{p} = \mathbf{m}$ [4], and this is the case for all ϕ_{inc} angles. Additionally, the element has 90-degree rotational symmetry, making it one possible realization of a simple zero-backscattering object

3. Simulated scattering from RHCP incidence field

How much error do we get when neglecting the loop polarisability? Whereas the scattered field magnitude for the chiral element had a close resemblance to a Huygens' pattern in [2] with linear incidence polarisation (LP), the simulations showed that the scattered axial ratio (AR) was generally elliptical with $AR \approx 0.5$ in the $-z$ direction. The chiral element was simulated with Ansoft HFSS. The particles were inserted next to each other, and the incidence field was RHCP. See Fig. 1(b) for geometry and dimensions. The results are seen in Fig. 2(a) and Fig. 2(b) for the scattering pattern and in Fig. 2(c) for the scattered AR. Compared to the LP incidence field, the element produces an order of magnitude deeper null in the backscattering direction and the scattered field is circular in the whole scattering hemisphere with $AR > -3$ dB. It turns out that some of the scattering components cancel each other with CP incident wave, resulting to purer scattering pattern.



(a) (b) (c)

Fig. 2: Simulated scattering directivity for the chiral element when the incident RHCP plane wave arrives at $\phi_{\mathrm{inc}} = 0^\circ$ and various θ_{inc}. (a) Parallel and (b) perpendicular theta plane relative to the incidence ϕ. (c) Scattered AR for normal incidence.

4. Discussion and Conclusion

Assuming operation at the first fundamental resonance of the particle (total length of the wire is $\lambda/2$) and the balance condition $Sk = l$, we can solve the relative dimensions of the particle. The length of the arm is $l \approx \lambda/7.46$ and the diameter of the loop is $D \approx \lambda/8.58$. Because the particles can be considered small compared to the wavelength, it makes sense to calculate the effective material parameters using the Maxwell Garnett approach for bi-anisotropic inclusions [5]. The result is expectedly the same as for a mixture of single chiral inclusions in x and y directions [5] (if neglecting the loop polarisability), and for the normal incidence the effective parameters are those of the host medium: $\mu_{\mathrm{eff}} = \mu$, $\epsilon_{\mathrm{eff}} = \epsilon$, and the chirality factor $\kappa = 0$. In fact, this hold for chiral particles of any dimensions as long as they are electrically small.

We have shown a realization of a small object composed of two canonical chiral particles which has very little backscattering, because the object is self dual with 90-degree rotational symmetry. We simulated backscattering for RHCP waves and saw that the loop polarisability has insignificant effect, making the self-dual approximation a good semi-analytical model. Of course, one does not need to use canonical chiral particles with separate wire and loop-antenna geometry. Theoretically, a simpler particle is the helical chiral particle, where even one particle produces CP scattered field [6].

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