Tuning interaction between metamaterial elements

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Abstract

We give an overview of the issue of coupling between metamaterial elements, an important factor in understanding and characterising their behaviour. We describe the well known Lagrangian model to describe this interaction. We show how the interaction coefficients can be calculated, and illustrate the importance of losses and fabrication errors in the tuning behaviour.

1. Introduction

The coupling between metamaterial elements is an important part of their response to the electromagnetic field, which complicates their description by simple models. The near-field coupling between neighbouring elements is a form of non-locality, which gives rise to spatial dispersion. From another perspective, if we modify the arrangement of metamaterial elements, we can tune this local field coupling, thus it can be regarded as in extra degree of freedom in the engineering of artificial composites. Many examples of such engineering can be found in the literature, see for example [1, 2] and references therein. The best starting point to describe coupling within a lattice is to consider a pair of neighbouring resonators. Although this is a much simpler system than the full lattice, it still exhibits a surprising degree of complexity, and contains much of the physics of the interaction problem.

2. Lagrangian description of a pair of resonators

A widely-used tool to describe a pair of resonators is the Lagrangian of a pair of coupled oscillators:

$$\mathcal{L} = \frac{L}{2}\left(\dot{Q}_1^2 + \dot{Q}_2^2 + 2\kappa_M \dot{Q}_1 \dot{Q}_2\right) - \frac{1}{2C}\left(Q_1^2 + Q_2^2 + 2\kappa_E Q_1 Q_2\right), \tag{1}$$

where $Q_n(t)$ represent the amplitudes of the fundamental modes of each resonator. We write it here using equivalent-circuit notation, but note that L and C should be interpreted as coefficients of stored energy for the fundamental mode of the resonator [2]. The coupling between the resonators has both electric κ_E and magnetic κ_M terms. After finding the corresponding dynamic equations, we find the following solutions for the symmetric and anti-symmetric modes:

$$\omega_S = \omega_0\sqrt{\frac{1+\kappa_E}{1+\kappa_M}} \qquad \omega_{AS} = \omega_0\sqrt{\frac{1-\kappa_E}{1-\kappa_M}}.$$

This shows that the frequency splitting between the two modes is determined by competition between the electric and magnetic interaction constants, which may be positive or negative depending on the relative arrangement of the metamaterial elements.

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3. Calculating the interaction constants

In order to rigorously calculate the interaction constants, we developed a method [2] based on the approximation that the hybridised modes are superpositions of the dominant modes in each resonator, with other modes being negligible. We start by noting that the terms in the Lagrangian correspond to the energy of the system. Since our resonant elements are closely spaced, by knowing the charge distribution $q(\mathbf{x})$ and current distribution $\mathbf{j}(\mathbf{x})$ we can find the energy in the quasi-static limit:

$$W_{E,mn} = \int_{V_m} \mathrm{d}^3x \int_{V_n} \mathrm{d}^3x' \frac{q(\mathbf{x})q(\mathbf{x}')}{4\pi\epsilon_0|\mathbf{x}-\mathbf{x}'|}, \tag{4}$$

$$W_{M,mn} = \int_{V_m} \mathrm{d}^3x \int_{V_n} \mathrm{d}^3x' \frac{\mu_0\, \mathbf{j}(\mathbf{x})\cdot\mathbf{j}(\mathbf{x}')}{4\pi|\mathbf{x}-\mathbf{x}'|}. \tag{5}$$

By comparing the relevant terms in the energy expressions, we can arrive at the the following expressions for the interaction constants

$$\kappa_M = \frac{W_{M,12}}{W_{M,11}} \qquad \kappa_E = \frac{W_{E,12}}{W_{E,11}}. \tag{6}$$

To illustrate the applicability of this method, we consider a pair of split rings in a broadside-coupled orientation, as shown in Fig. 1. We increase the offset δ_a between them, and plot the corresponding resonant frequencies. It can be seen that the full numerical results (background colour) agree very well with the results from the Lagrangian model (lines).

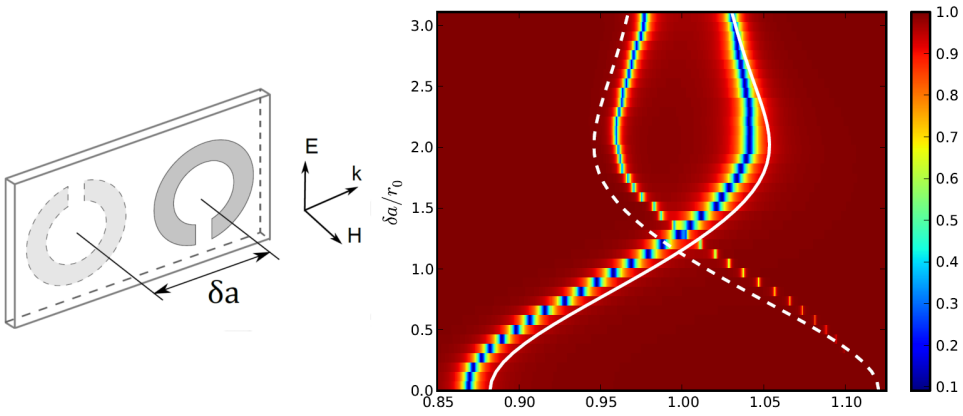

Fig. 1: Resonant frequencies of a pair of rings as a function of offset between their centres, comparing numerical results with those obtained from a Lagrangian model (solid - symmetric, dashed - antisymmetric.

4. The influence of losses and fabrication imperfections

In the experiments there will always be some small difference in the resonators due to fabrication imperfections. In addition, the theory developed so far has not included the influence of losses, neglecting the typically strong radiation from the resonators, as well as ohmic dissipation. To illustrate the effect on coupling, we consider a system of a pair of split-ring resonators (SRRs) rotated by angle θ about their common axis, as reported in [3]. We modify our system of equations by introducing a dissipation coefficient Γ and a detuning parameter $\delta\omega$. This results in the the following dispersion equation for the

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modes:

$$D(\omega,\theta) = \left[(\omega_0+\delta\omega)^2 + j2\Gamma\omega - \omega^2\right]\left[(\omega_0-\delta\omega)^2 + j2\Gamma\omega - \omega^2\right] - \left[\kappa_E(\theta)\omega_0^2 - \kappa_M(\theta)\omega^2\right]^2 = 0 \quad (7)$$

Using the theory of Morse critical points [4], we can study the properties of the function $D(\omega,\theta)$ as we increase the loss parameter Γ. As illustrated in Fig. 2, for low values of Γ the detuning $\delta\omega$ causes a repulsion of the modes. However, beyond some critical value of Γ, the crossing of modes is restored. Thus losses are very important for understanding behaviour at critical points of the tuning dispersion. For full details, see [3].

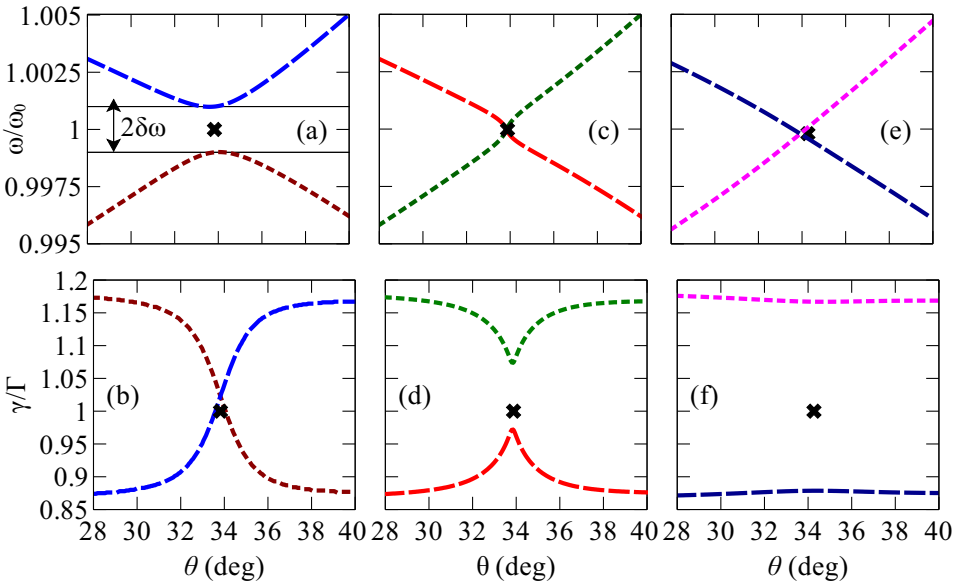

Fig. 2: (a) Resonant frequencies and (b) absorption coefficients of a pair of coaxial rings when $\Gamma = 1\times10^{-3}\omega_0$, (c) Resonant frequencies and (d) absorption coefficients when $\Gamma = 7\times10^{-3}\omega_0$, showing that losses restore the crossing. (e) Resonant frequencies and (f) absorption coefficients when losses are increased to $\Gamma = 2\times10^{-2}\omega_0$.

5. Conclusion

We have given an overview of coupling between metamaterial elements, and shown how this can be described with a Lagrangian model. We presented a method for calculating the interaction coefficients, and showed that losses and fabrication errors can have a significant impact on the tuning behaviour at critical points of the dispersion curve.

References

[1] H. Liu, Y.M. Liu, T. Li, S.M. Wang, S.N. Zhu, and X. Zhang, Coupled magnetic plasmons in metamaterials, *Physica Status Solidi (B)*, vol. 246, pp. 1397-1406, 2009.

[2] D. Powell, M. Lapine, M.V. Gorkunov, I.V. Shadrivov, and Y. Kivshar, Metamaterial tuning by manipulation of near-field interaction, *Physical Review B,* vol. 82, p. 155128, 2010.

[3] D.A. Powell, K. Hannam, I.V. Shadrivov, and Y.S. Kivshar, Near-field interaction of twisted split-ring resonators, arXiv:1101.5454v1

[4] A.B. Yakovlev and G.W. Hanson, Mode-transformation and mode-continuation regimes on waveguiding structures, *IEEE Transactions on Microwave Theory and Techniques,* vol. 48, pp. 67-75, 2000.

