Artificial magnetism and modes at millimeter waves in 3D lattices of titanium oxide microspheres

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Abstract

We show a study of complex modes and the artificial magnetic properties at millimeter waves of 3D-periodic arrays of microspheres made of high permittivity materials, specifically titanium oxide, embedded in a homogeneous background. Each microsphere is modeled as a single magnetic dipole and complex modes in the 3D lattice are computed by means of the 3D-periodic dispersion relation including mutual interactions. The effective permeability and the effective refractive index versus frequency obtained by the modal analysis are shown and compared to those obtained with Maxwell Garnett homogenization theory and also to those retrieved by scattering parameters of finite thickness structures obtained through HFSS simulations by using the Nicolson-Ross-Weir method.

1. Introduction

Arrangements of 3D-periodic arrays of microspheres made of high permittivity materials can be engineered to obtain artificial bulk magnetism and double negative materials. Mode analysis of 3D-periodic arrays of dielectric spheres, and the computation of their effective parameters have been analyzed in [1-4] at gigahertz and infrared frequencies. At terahertz frequencies, titanium oxide (TiO_2) has high permittivity. In this paper, we analyze the complex modes in the 3D-periodic arrays of TiO_2 microspheres in Fig. 1 for transversal (with respect to the mode traveling direction, T-pol) polarization, though in certain frequency ranges also longitudinal modes are present (not shown here). The effective permeability and the effective refractive index obtained by the modal analysis for transverse polarization are compared to those obtained by Maxwell Garnett (MG) homogenization theory [5] and to those obtained through full-wave simulations and the Nicolson-Ross-Weir (NRW) method [6].

2. Modes determination

Single dipole approximation (SDA) [7] is adopted to model each small microspheres (with respect to the wavelength) when the magnetic effects are considered to be dominant with respect to the electric ones. According to SDA, the induced dipole moment in a microsphere is $\mathbf{m} = \alpha_{\mathrm{mm}} \mathbf{H}^{\mathrm{loc}}$, with $\alpha_{\mathrm{mm}} = 6\pi i b_1 / k^3$ being its magnetic polarizability according to Mie theory [8], b_1 is the magnetic Mie dipole scattering coefficient, $k = \sqrt{\varepsilon_h} k_0$ is the host wavenumber, k_0 is the free space wavenumber,

ISBN 978-952-67611-0-7

and $\mathbf{H}^{\text{loc}}$ is the local field produced by all the microspheres of the array, except the considered microsphere, plus the external incident field to the array. Complex modes in 3D arrays are found by using the procedure in [7] adapted to the case of magnetic fields.

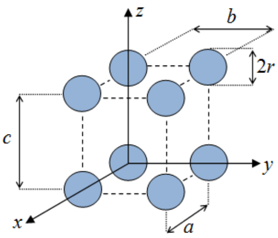

Fig. 1: 3D-periodic array of TiO_2 (titanium oxide) microspheres embedded in a homogeneous medium with permittivity ε_h. The large permittivity of each microsphere is ε_m. The radius of each microsphere is *r*; *a*, *b* and *c* are the periodicities along *x*-, *y*- and *z*-direction, respectively.

3. Dispersion diagrams and effective parameters

We analyze the complex modes traveling along the *z*-direction in a 3D-periodic array of TiO_2 microspheres in free space (i.e., $k = k_0$), for T-pol in the frequency range between 250 GHz and 350 GHz, in which the magnetic effects are dominant. In the analyzed frequency range, the relative permittivity of each TiO_2 microsphere is assumed to be constant and equal to $\varepsilon_m = 94 + i2.35$ [9]. The radius of each microsphere is $r = 52\ \mu\text{m}$, and we assume a cubic lattice with $a = b = c = 126\ \mu\text{m}$, thus the filling factor is $f = 4\pi r^3 / (3a^3) = 0.294$. The complex modes for T-pol are shown in Fig. 2. Physical modes traveling along positive *z* excited by a source at $z = 0$, for instance, have $\alpha_z \geq 0$, and travel long distances if $\alpha_z << k$ (low decay); it follows that 'Mode 1' is the dominant mode and 'Mode 2' has a higher attenuation constant. We derive the effective parameters neglecting 'Mode 2'.

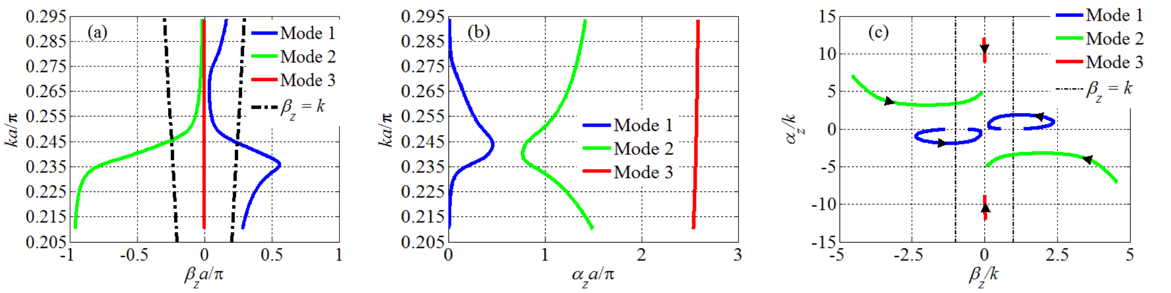

Fig. 2: Guided wavenumber-frequency dispersion diagram for T-pol. (a) Real part and (b) imaginary part of the wavenumber $k_z = \beta_z + i\alpha_z$. Notice that only the physical modes in the I and II quadrant of the complex k_z plane are shown. (c) Complex plane evolution; arrows indicate direction of increasing frequency.

The comparison of the effective parameters computed through the three aforementioned methods is reported in Figs. 3 and 4. The effective refractive index obtained by the wavenumber of the main propagating mode (assuming a medium of magnetic dipoles) is $n_{\text{m}}^{\text{eff}} = k_z / k$ and the effective permeability is $\mu^{\text{eff}} = (n_{\text{m}}^{\text{eff}})^2$. In the MG formulation we have considered both the electric and magnetic polarizabilities obtained from Mie theory. With the full-wave simulation we obtained transmittance and reflection pertaining to eleven layers of arrayed TiO_2 microspheres, stacked in the direction of propagation, and the effective parameters are retrieved by using the NRW method. Notice the good agreement of the effective permeability and index in the entire frequency range retrieved from all the three methods. The refractive index result from modal analysis in Fig. 3 takes into account the contribution from the background, non-resonant permittivity ε^{eff} created by the dielectric spheres, as calculated with MG: $n^{\text{eff}} = \sqrt{\varepsilon^{\text{eff}}}\, n_{\text{m}}^{\text{eff}}$.) It can be stated that artificial magnetism can be achieved by the periodic array in Fig. 1.

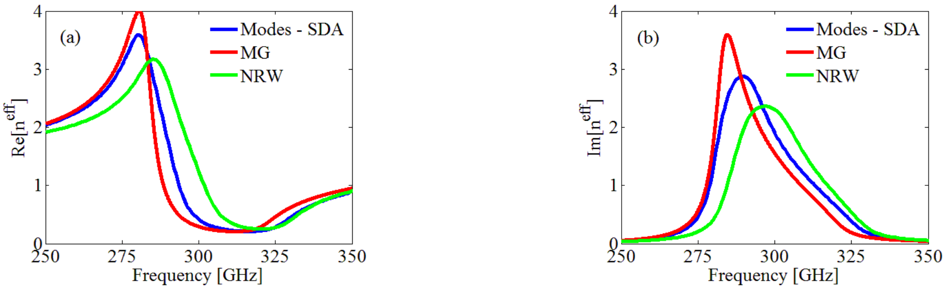

Fig. 3: (a) Real part and (b) imaginary part of the effective refractive index computed by means of the 3D-periodic dispersion relation (Modes - SDA), Maxwell Garnett approximation (MG) and Nicolson-Ross-Weir retrieval method (NRW).

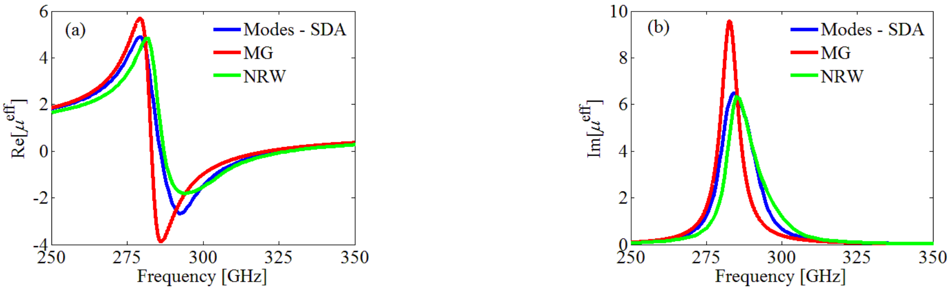

Fig. 4: As in Fig. 3, for the effective permeability.

4. Conclusion

A description of complex modes in 3D-periodic arrays of TiO_2 microspheres has been provided. Effective parameters obtained by a modal analysis have been compared to the ones obtained by Maxwell Garnett and by NRW retrieval method through full-wave simulations of finite layers. It has been shown that artificial magnetism can be achieved by such a periodic array. Simulations show that large values of permeability as well as values close to zero and negative ones can be achieved.

Acknowledgment – This work has been partly funded by the European Union’s FP7-NMP-2008 program under the METACHEM project (grant n^{o} 228762) and the French GIS “Advanced Materials in Aquitaine”.

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