

# On the implementation of a robust algorithm which automates the synthesis of artificial transmission lines based on CSRRs

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## Abstract

Recently, the authors of this paper have been able to automatically generate the layout for microstrip lines loaded with CSRRs [1]. The method proposed in that paper, is working nowadays successfully for composite right/left-handed (CRLH) and negative permittivity unit cell lines [2] (see Fig. 1). Several improvements have been introduced, and summarized in the present contribution.

## 1. Introduction

Since metamaterials were first investigated, very different research lines have emerged in the scientist community based on applying those concepts into different branches of knowledge such as acoustics, terahertz technologies and nano-photonics. Our interest is focused on the microwave applications in planar technology, being more precise on the resonant approach of artificial transmission lines. Recently, the authors of this paper have been able to automate the generation of the layout for microstrip lines loaded with CSRRs [1], which otherwise is a complex task in time and resources.

The method proposed in that paper, is working successfully for both unit cells shown in Fig.1 and reported in [2], which are a left-handed media and a negative permittivity line. After testing the program intensively, several improvements have been introduced, and summarized in the present article.

## 2. Improved algorithm

Basically the core of the algorithm is based on aggressive space mapping techniques (ASM) as in previous publication. Given an ideal target frequency response, characterized by the vector  $x_c^*$  (which contains the electrical parameters of the equivalent circuit model), a final synthesis  $x_{em}$  is achieved by means of the iterative optimization of the equivalent circuit model, see Fig.1.

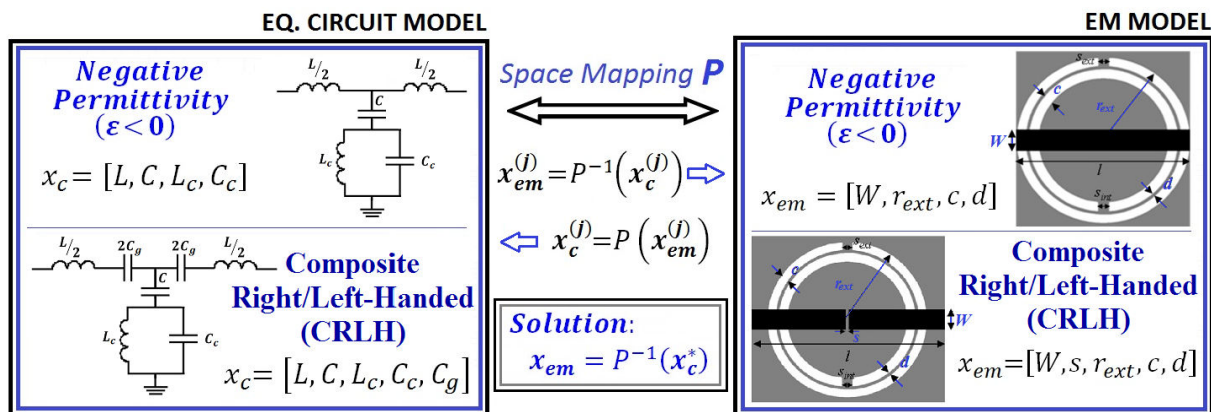


Fig. 1: Conceptual diagram of space mapping techniques for the cells synthesized.

The number of terms considered in both models are the same (physical dimensions used with the EM simulator and lumped elements of the equivalent circuit model), in order to have a square Broyden matrix. This matrix,  $\mathbf{B}^{(j)}$ , is used during the ASM algorithm to obtain the new step  $\mathbf{h}^{(j)}$ , which is approaching the solution iteratively according to:

$$\mathbf{x}_{em}^{(j+1)} = \mathbf{x}_{em}^{(j)} + \mathbf{h}^{(j)} = \mathbf{x}_{em}^{(j)} + (-\mathbf{B}^{(j)})^{-1} \cdot \mathbf{f}^{(j)} \quad (1)$$

Convergence is achieved when the norm of the error function  $\mathbf{f}^{(j)}$  is smaller than a fixed value  $\eta$ . The whole flow diagram is shown in Fig.2. Three different parts are enhanced: initiation, constrained and Line Search (LS). The changes introduced in the first two phases are commented here, whereas the Line Search stage will be explained in section 3, since it was not present in the initial approach.

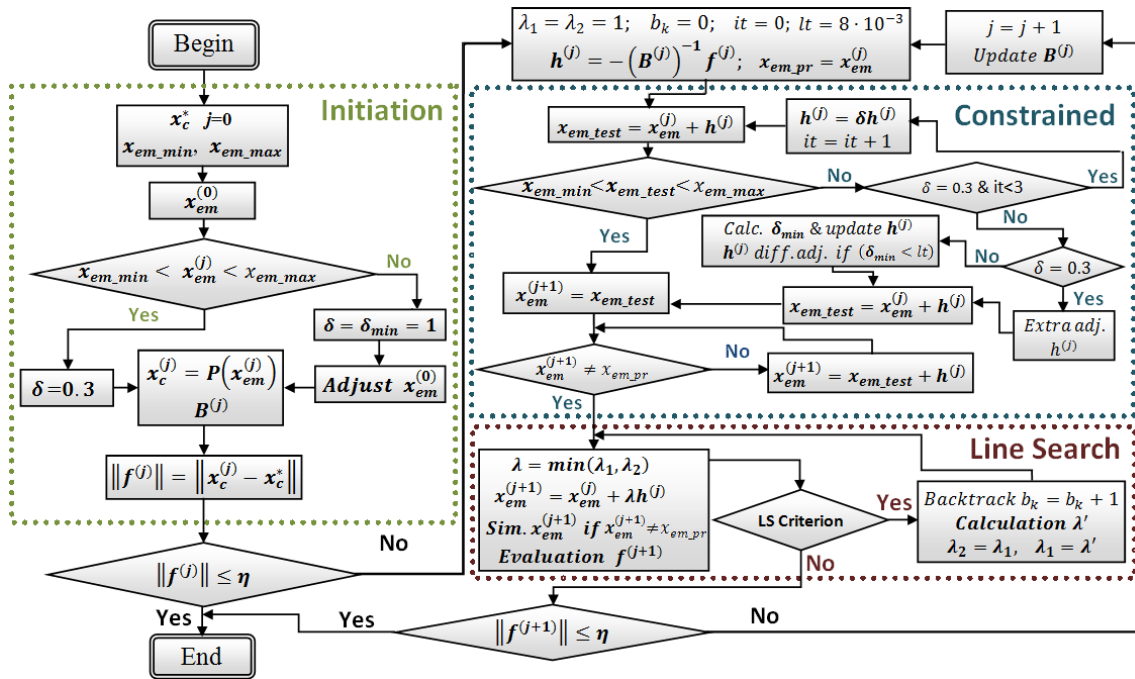


Fig. 2: Improved algorithm based on ASM in combination with LS

During the initiation stage, a first geometry of the cell,  $\mathbf{x}_{em}^{(0)}$ , is calculated. Analytical models (classical expressions for microstrip lines, CSRR and gap) are used, but they are suitable for modelling isolated components, so just useful to obtain an initial approximation of the synthesis. If these dimensions are not within the validation limits, they are rounded to the closest values. When that happens, using a fixed shrinking factor  $\delta$  for the constrained approach turned not to be the best option. Using instead  $\delta_{min}$  which is the minimum factor requested for obtaining dimensions inside the valid range, leads into a fastest convergence as it can be seen in the example of Fig. 3.

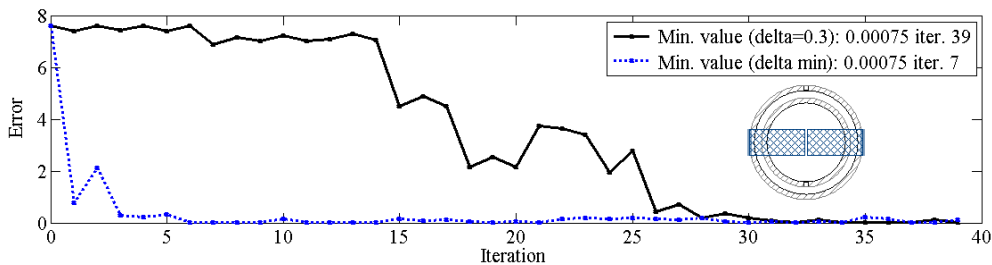


Fig. 3: Evolution of the error  $\|\mathbf{f}^{(j)}\|$  with ASM constrained using different delta: fixed ( $\delta=0.3$ ) or variable ( $\delta_{min}$ ), for a target response given by:  $L[\text{nH}]=5.141$ ,  $L_c[\text{nH}]=2.635$ ,  $C[\text{pF}]=14.841$ ,  $C_c[\text{pF}]=2.812$ ,  $C_g[\text{pF}]=0.773$ .

Furthermore, if the size of  $\mathbf{h}^{(j)}$  is too small, and as result the new solution is roughly equal to the previous iteration synthesis, i.e.  $\mathbf{x}_{em}^{(j)}$ , the step is added several times to the synthesis till at least one of the dimensions varies. Otherwise, the matrix  $\mathbf{B}$  is ill conditioned and brings instability to the algorithm. To

avoid this fact, the shrinking factor is also considered to be in certain range and if not an extra adjusting is made. When  $\delta$  is fixed, the number of times that  $\delta$  is applied to  $\mathbf{h}^{(j)}$  is limited by  $it$ . In the case of  $\delta_{\min}$ , if the calculated value is under  $it$ , a bigger factor is used and  $\mathbf{h}^{(j)}$  is readjusted afterwards.

### 3. Line Search

The line search (LS) technique is introduced to avoid abrupt incremental variations of the error  $\|\mathbf{f}^{(j)}\|$ , that ASM algorithm can present. Introducing LS makes that  $\|\mathbf{f}^{(j)}\|$  follows a global decreasing behaviour till achieving convergence. An example where LS results in a clear improvement is shown in Fig. 4. Those cases where ASM does not have important variation, or the initial layout has a big error, LS can derive in slow convergence. To avoid it, a modified criterion of applying LS is proposed.

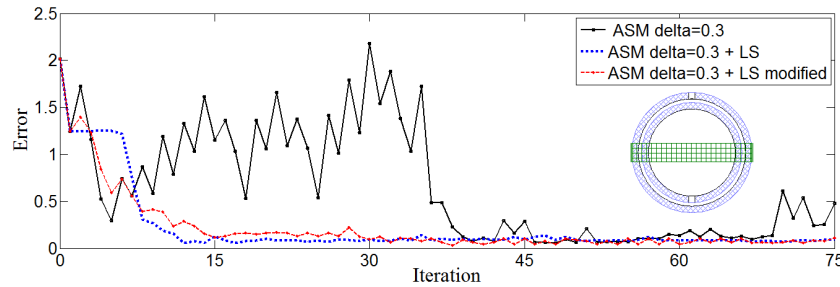


Fig. 4: Evolution of error for a target response given by  $L[\text{nH}]=4.90$ ,  $C[\text{pF}]=2.13$ ,  $L_c[\text{nH}]=2.30$ ,  $C_c[\text{pF}]=2.76$ .

When the error function is remarkably increased with respect to the previous iteration, taking the full Newton step  $\mathbf{h}^{(j)}$  is a bad choice. The length of the step is controlled with LS in the same Newton direction by means of a factor  $\lambda$ . First time,  $\lambda$  is equal to unit (i.e. the normal evolution of the algorithm), and afterwards is calculated as indicated in [3]. The criterion to apply LS is given by (2):

$$f^{(j+1)} \geq f^{(j)} + \kappa \cdot \nabla f^{(j)} \cdot (x_{em}^{(j+1)} - x_{em}^{(j)}), \quad \text{with } \kappa = 10^{-4} \quad (2)$$

Different backtracks along  $\mathbf{h}^{(j)}$  direction are applied, till the error decreases or  $x_{em}^{(j+1)}$  coincides with  $x_{em}^{(j)}$ . In the case of negative permittivity cell, LS criterion is modified and only one backtrack is made until the error is under 0.5. In CRLH cells, the error is larger and the normalized error (3) must be taken into account (due to the fact that  $C$  is much bigger than the rest of the parameters considered).

$$\|f_{norm}^{(j)}\| = \sqrt{\left(1 - \frac{L}{L^*}\right)^2 + \left(1 - \frac{C_g}{C_g^*}\right)^2 + \left(1 - \frac{C}{C^*}\right)^2 + \left(1 - \frac{C_c}{C_c^*}\right)^2 + \left(1 - \frac{L_c}{L_c^*}\right)^2} \quad (3)$$

Hence, to apply LS, condition (2) must be satisfied and the normalized error decreased respect to the previous iteration, being the number of backtracks limited to one until the absolute error is under 2.5.

### 4. Conclusion

This improved algorithm is more robust than the previous proposed version, having increased the convergence rate and accuracy for different cases. The introduction of Line Search has not always provided as positive effects as expected; since the convergence can become slower. However, different slight modifications of LS have been introduced to reduce that additional computational effort.

### References

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