Nonlinear modes and wave transmission in SQUID-based metamaterials

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Abstract

A lattice of rf-driven superconducting quantum interference devices forms a metamaterial that may respond resonantly with a positive or negative effective magnetic permeability. When the lattice is driven in a fully nonlinear regime, nonlinear modes may be generated in the form of intrinsically localized modes. Transmission studies show the appearance of multistability and chaos. The resulting system is a complex quantum metamaterial with exciting dynamical properties that may lead to interesting applications.

1. Introduction

An extended nonlinear metamaterial may be considered as a complex dynamical system with a wealth of interesting properties [1]. Metamaterials are metacrystals made of units that have dimensions larger than those of atoms and molecules while each unit has specific functional properties. Prototypical linear metamaterials are made of SRR's (split ring resonators), each forming an RLC circuit and leading electromagnetic wave propagation with left-handed properties [2]. The use of rf SQUID's (superconducting quantum interference device) in place of SRR's leads to the formation of superconducting metamaterials with inherently nonlinear properties. The resulting complex medium encompasses discreteness, periodicity as well as nonlinearity, while the generation of various spatiotemporal nonlinear modes may lead to some form of adaptivity [3].

An rf SQUID is a closed driven superconducting circuit that consists of a Josephson junction together with capacitance, inductance and some resistance. Changes in the driving frequency of the external magnetic field threaded through the circuit leads to different types of responses; this feature is taken in advantage when a lattice of SQUIDs is constructed forming a SQUID metamaterial. In the effective medium approximation the specific interaction of adjacent units is not taken into account, leading that to a global response from the medium [2]. When the inductive coupling of nearby units is also considered the properties of the medium become more interesting due to the interplay of discreteness with unit dynamical properties that are further enhanced by the Josephson junction nonlinearity. In this latter case we observe the appearance of intrinsically localized modes (discrete breathers) that are dynamical self-localized modes induced by the driver field in various lattice locations [3]. These modes bear some resemblance to the discrete breather states found experimentally in Josephson junction ladders [4,5]. The generation of dynamic inhomogeneities of this form, turns the SQUID meta-lattice to a spatiotemporally complex dynamical system with novel properties [6].

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2. Effective medium rf SQUID equation

The dynamics of a single rf SQUID driven by an external magnetic field is given by the following nonlinear equation (in normalized variables)

$$\frac{d^2 f}{d\tau^2} + \gamma \frac{df}{d\tau} + \beta \sin(2\pi f) + f = f_{ext} \tag{1}$$

where f is the normalized flux through the SQUID, γ is related to the resistance induced damping, τ is time normalized to linearized frequency, β is related to the critical Josephson current while f_{ext} is the normalized flux from the external magnetic field [2]. The eq. (1) has rich dynamical properties depending on the specific regime of study; in the effective medium approximation describes the collective response of the SQUID lattice leading to the appearance of both right-handed as well as left-handed response in a resonant regime (Fig. 1).

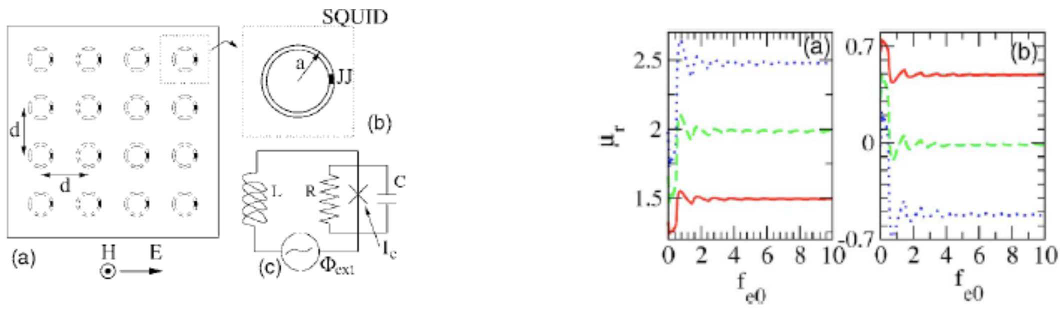

Fig. 1 Left panel: (a) SQUID lattice, (b) Individual SQUID, (c) RCSJ approximation. Right panel: Effective magnetic permeability for the SQUID metamaterial as a function of the normalized driving amplitude. Driving frequency slightly below resonance, in (a) and slightly above in (b) while colour lines depict different filling fractions [2]. Note that permeability may be positive or negative.

3. Intrinsic nonlinear localization in an rf SQUID lattice

When we consider a two dimensional square lattice of SQUID elements and drive it externally with a magnetic field the mutual inductance among the rings introduces dipole-dipole coupling. If we retain only the nearest neighbor SQUID interaction we may replace Eq. (1) for the single unit with the following normalized set:

$$\frac{d^2 f_{nm}}{d\tau^2} + \gamma \frac{df_{nm}}{d\tau} + f_{nm} + \beta \sin(2\pi f_{nm}) - \lambda_x (f_{n-1,m} + f_{n+1,m}) - \lambda_y (f_{n,m-1} + f_{n,m+1}) = [1 - 2(\lambda_x + \lambda_y)] f_{ext} \tag{2}$$

where f_{nm} is the normalized flux passing from the (n,m) ring while λ_x, λ_y are the nearest neighbour couplings for the inductively coupled rings. In the fully nonlinear regime the on-site potential for a single SQUID is

$$u_{SQ} = \frac{1}{2}(f - f_{ext})^2 - \frac{\beta}{2\pi}\cos(2\pi f) \tag{3}$$

This form of nonlinear potential leads naturally in the weak-coupling limit to the formation of intrinsically localized modes (discrete breathers) localized in a small neighbourhood of the lattice (Fig. 2). In this case, while the rest of the lattice is in a given flux or current response state, in the localized region the system responds in a drastically different way.

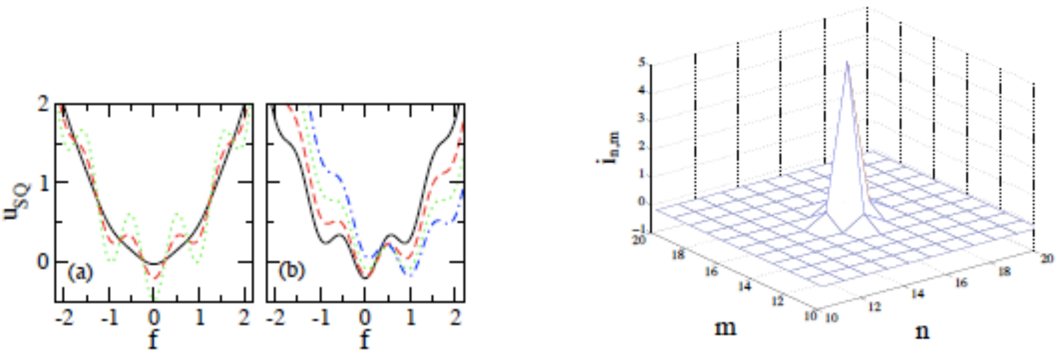

Fig. 2: Left panel: On-site potential for single SQUID for (a) fext=0, β=0.15 (solid), 1.27 (dashed), 3 (dotted) (b) β=1.27 with f_{ext}= 0 (solid), 0.25 (dashed), 0.5 (dotted), 0.75 (dashed-dotted). Right panel: Snapshot of a dissipative discrete breather at its maximum (induced current as a function of the two lattice directions).

4. Conclusion

An rf-driven SQUID is an intrinsically nonlinear unit that shows typical nonlinear features such as bistability and hysteresis. When a lattice of coupled SQUIDs is formed the resulting two dimensional system behaves as a complex metamaterial showing a wealth of interesting features. The SQUID lattice responds both in a right- and left-handed way while discrete breathers may form in various system locations. The latter are periodically varying space-localized modes with distinct spectral features and are induced due to nonlinearity in the translationally invariant metamaterial. Wave transmission properties in small SQUID lattices show the presence of multistability and chaos [7].

References

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