Proxiton-polaritons on metamaterial slabs of coupled split rings

E. Shamonina

Optical and Semiconductor Devices Group, Electrical and Electronic Engineering (EEE) Department, Imperial College, London SW7 2BT, UK, e-mail: e.shamonina@imperial.ac.uk

Abstract

In split-ring metamaterials, inter-element interactions result in propagation of magnetoinductive waves. These slow waves can couple to and influence the propagation of electromagnetic waves forming so-called *proxiton-polaritons*, similar to plasmon-polaritons in a bulk metal. Using our effective-medium model accounting for both bulk and surface proxitons-polaritons, we are able to design SRR-based near field manipulating devices for TE polarisation, in full analogy to a silver slab operating as a near-perfect lens for TM polarised light.

1. Introduction

Effective-medium model for split-ring metamaterials has shown, disregarding interactions between individual elements, that in the absence of losses the effective permeability tends to plus/minus infinity as the resonant frequency is approached from left or right [1]. A microscopic description of split-ring metamaterials, that takes inter-element interactions into account, was proposed in [2]. A consequence of these interactions is the emergence of slow magnetoinductive waves (which have also been referred to more recently as magnetization waves [3] and magnetic plasmons [4]) unaccounted for in a simplified effective medium theory. These slow waves are eigenmodes of the metamaterial and can be expected to couple to and influence the propagation of electromagnetic waves forming so-called *proxiton-polaritons*, similar to plasmon-polaritons in a bulk metal.

A 1D theory incorporating proxiton-polaritons into the effective-medium model was derived in [5]. An extension to an isotropic 3D case was devised in [6] followed by a further extension that included retardation effects as well [7] and by generalization to an anisotropic case [8]. Our model [5,8] establishes the relationship between effective permeability and circuit characteristics of a generally anisotropic metamaterial structure. Key conclusions from the model are that due to inter-element coupling (i) the range of values where the permeability becomes negative may be strongly shifted away from the resonant frequency and (ii) the effective permeability exhibits strong spatial anisotropy and spatial dispersion. In a subsequent work we extended our study to properties of surface modes of proxiton-polaritons at an interface between vacuum and an anisotropic metamaterial [9].

In this paper we explore properties of coupled surface proxiton-polaritons on an anisotropic metamaterial slab comprising split rings. We demonstrate on a number of examples how our model can be used for the design of split-ring based near-field lenses for the desired frequency range.

2. Coupled surface proxiton-polaritons on a metamaterial slab

We consider a 2D rectangular array of split rings consisting of two planar-axial sub-lattices as shown in Fig. 1a and a TE electromagnetic wave propagating in the *xz* plane. The corresponding distributed *LC* transmission line is shown in Fig.1b. Here we use L_t, C_t, $J_x(m,n)$ and $J_z(m,n)$ as the inductance, the capacitance and the *x* and *z* current components in the unit cell (m,n) attributed to the transverse electromagnetic wave. L, C, $I_x(m,n)$ and $I_z(m,n)$ are the self-inductance, the capacitance and the currents attributed to the ring resonators in both sub-lattices. $M'_{x,z}$ are the mutual inductances responsible for the coupling of the electromagnetic wave to the split rings of the corresponding sub-lattices, and $M_{\mathrm{a}x}, M_{\mathrm{p}x}, M_{\mathrm{a}z}$ and $M_{\mathrm{p}z}$ are the mutual inductances between neighbouring elements of either of the sub-lattices in either the axial or in the planar arrangement.

ISBN 978-952-67611-0-7

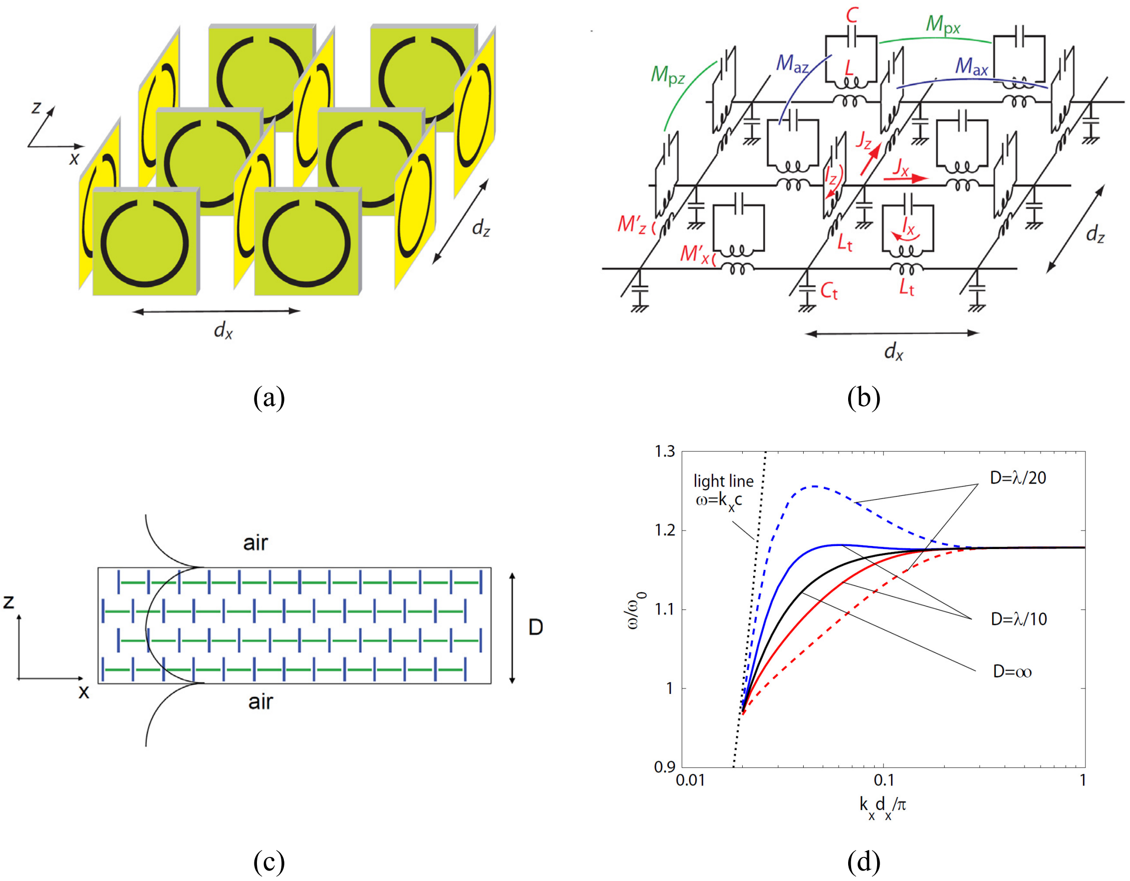

Fig. 1: (a) A metamaterial consisting of two sub-lattices of split rings; (b) its equivalent circuit model; (c) top view of a slab of a brick-wall SRR metamaterial; (d) surface proxiton-polariton dispersion for various thicknesses D of the brick-wall slab. $\kappa_{\rm ax} = 0.1$, $\kappa_{\rm px} = 0.1, \kappa_{\rm az} = \kappa_{\rm pz} = 0$, $q_x^2 = q_z^2 = 0.56$, $d_x = d_z = \lambda/100$.

The transfer function of such a metamaterial slab of the thickness D surrounded by air on both sides (see Fig.1c), for an incident electromagnetic wave with the k_x component along the boundary, can be obtained in the form

$$T = \frac{4\varsigma}{(1+\varsigma)^2 \exp(\mathrm{i}k_{z,\mathrm{slab}}D) - (1-\varsigma)^2 \exp(-\mathrm{i}k_{z,\mathrm{slab}}D)}, \tag{1}$$

with $\varsigma = \mu_x k_{z,\mathrm{air}} / k_{z,\mathrm{slab}}$, $(k_{z,\mathrm{air}})^2 + k_x^2 = \omega^2 / c^2$, $\mu_z (k_{z,\mathrm{slab}})^2 + \mu_x k_x^2 = \mu_x \mu_z \omega^2 / c^2$. μ_x and μ_z are the components of the spatially dispersive anisotropic permeability tensor expressed in terms of the circuit parameters,

$$\mu_{x,z} = 1 - q_{z,x}^2 \left(1 - \frac{\omega_0^2}{\omega^2} + 2\frac{M_{\mathrm{ax,az}}}{L}\cos k_{x,z} d_{x,y} + 2\frac{M_{\mathrm{pz,px}}}{L}\cos k_{z,x} d_{z,x}\right)^{-1}, \tag{2}$$

with $\omega_0^2 = 1/LC$, $\omega_t^2 = 1/L_t C_t$ and $q_{x,z}^2 = (M'_{x,z})^2 / LL_t$. In the absence of magnetoinductive coupling, Eq (2) reduces to the known result provided by the simplified effective-medium theory [1].

The dispersion equation of coupled surface proxiton-polaritons follows from the condition that the denominator in Eq (1) is zero. Performing the calculations we obtain the following two solutions reminiscent to those for surface plasmons on a silver slab (see e.g. [10])

$$\varsigma = -\tanh(\mathrm{i}k_{z,\mathrm{slab}}D/2) \quad \text{and} \quad \varsigma = -\coth(\mathrm{i}k_{z,\mathrm{slab}}D/2)\,. \tag{3}$$

Taking as an example a 'brick-wall' structure of thickness D [9] shown in Fig.1c with elements coupled only in the z direction, we obtain dispersion curves for the coupled surface modes shown in Fig.1d for D varying from λ/20 to infinity.

In the lecture we shall present further examples and systematise conditions for subwavelength imaging, identifying split-ring configurations suitable (as well as those not suitable) for subwavelength imaging via surface modes. Using realistic parameters we shall show how the transfer function of a split-ring slab depends on the unit cell structure, slab thickness, and losses. Considering that surface waves of various kinds have found a wide range of applications in the past, it is envisaged that surface proxiton-polaritons will open up fresh possibilities. The model, with some minor modification, can be applied to the description of other configurations as well. If, e.g., in addition, we include the loading of the transmission line by an inductance, corresponding to the rods, we can model negative permittivity as well resulting in a negative index material.

3. Conclusion

A circuit model for split-ring metamaterials with strong inter-element coupling is used to describe proxiton-polaritons, hybrid modes of interaction of magnetoinductive and electromagnetic waves. Conditions for coupled modes of surface proxiton-polaritons for a slab are identified, considering that the effective permeability of a split-ring metamaterial exhibits strong spatial dispersion and anisotropy. The role of various parameters, such as the unit cell structure, slab thickness, and losses, is investigated.

Acknowledgments: *Financial support of the Leverhulme Trust and of the DFG is gratefully acknowledged.*

References

[1] J.B. Pendry, A.J. Holden, D.J. Robbins and W.J. Stewart, Magnetism from conductors and enhanced nonlinear phenomena, *IEEE Trans. Microw. Theory Techn.* 47, 2075 (1999).

[2] E. Shamonina, V.A. Kalinin, K.H. Ringhofer and L. Solymar, Magnetoinductive waves in one, two, and three dimensions, *J. Appl. Phys.* 92, 6252 (2002).

[3] G. Dolling, M. Wegener, A. Schädle, S. Burger, and S. Linden, Observation of magnetization waves in negative-index photonic metamaterials, *Appl. Phys. Lett.* 89, 231118 (2006).

[4] H. Liu, D. A. Genov, D. M. Wu, Y. M. Liu, J. M. Steele, C. Sun, S. N. Zhu, and X. Zhang, Magnetic plasmon propagation along a chain of connected subwavelength resonators at infrared frequencies, *Phys. Rev. Lett.* 97, 243902 (2006).

[5] R.R.A. Syms, E. Shamonina, V. Kalinin and L. Solymar, A theory of metamaterials based on periodically loaded transmission lines: Interaction between magnetoinductive and electromagnetic waves, *J. Appl. Phys.* 97, 064909 (2005).

[6] J.D. Baena, L. Jelinek, R. Marqués and M. Silveirinha, Unified homogenization theory for magnetoinductive and electromagnetic waves in split-ring metamaterials. *Phys. Rev. A* 78, 013842 (2008).

[7] M. G. Silveirinha, J. D. Baena, L. Jelinek and R. Marques, Nonlocal homogenization of an array of cubic particles made of resonant rings, *Metamaterials* 3, 115 (2009).

[8] A. Radkovskaya, E. Tatartschuk, C.J. Stevens, D.J. Edwards, E. Shamonina and L. Solymar, Magnetic plasmon-polaritons in split-ring metamaterials with strong coupling, 3rd International Congress on Advanced Electromagnetic Materials in Microwaves and Optics Metamaterials'2009, London, 2009.

[9] E. Shamonina and A. Radkovskaya, Surface and bulk proxiton-polaritons in split-ring metamaterials, 4th International Congress on Advanced Electromagnetic Materials in Microwaves and Optics Metamaterials'2010, Karlsruhe, 2010.

[10] L. Solymar and E. Shamonina, Waves in Metamaterials (Oxford University Press, Oxford, 2009).