Perfect lens based on ideal phase conjugating surfaces

S. I. Maslovski[1], **Y. Rapoport**[2,3], **and S. A. Tretyakov**[2]

[1] Departmento de Engenharia Electrotécnica, Instituto de Telecomunicações
Universidade de Coimbra, Pólo II, 3030 Coimbra, Portugal; e-mail: stas@co.it.pt

[2] Department of Radio Science and Engineering
Aalto University School of Electrical Engineering
PO Box 13000, FI-00076, Aalto, Finland
Fax: +358–947022152; email: sergei.tretyakov@aalto.fi

[3] Kiev Taras Shevchenko National University, Astronomy and Space Physics Department, Prospect Glushkov, 2, Physics Faculty, 03680 Kiev, the Ukraine

Abstract

It is known that two parallel sheets with the complex-conjugation transition condition act as a perfect lens, mimicking all the properties of a slab filled with an isotropic medium with permittivity and permeability both equal to -1. In this presentation we will discuss the physical meaning of the phase conjugating boundary conditions and outline possible approaches to practical realization of this superlens.

1. Introduction

The *perfect lens* [1] is a device which focuses the field of a point source into a point, that is, the perfect lens focuses both propagating and evanescent fields. It is known [1] that a planar slab of a material with the relative permittivity and permeability both equal to -1 has the perfect-lens properties. Practical realization of such materials is, however, a significant challenge, especially for optical frequencies, and any realization will suffer from some imperfections. In 2003, it was shown [2] that two parallel sheets with phase conjugating boundary conditions for tangential fields on the two sides of the sheets also have the properties of the perfect lens. While in theory it is easy to prove that the system indeed operates as a perfect lens, assuming that the phase-conjugating boundary conditions hold, the question arises what is the physical meaning of these conditions and if and how such sheets can be possibly realized. This question will be addressed in this presentation.

2. Theory and design

Let us start from outlining the idea from our year 2003 paper [2]. Consider an ideal Veselago lens depicted in Figure 1. Let the relative permittivity and permeability of the medium surrounding the lens be equal to 1 and the relative parameters of the lens material to -1 at the working frequency ω, respectively. The boundary conditions at the lens interfaces are the usual continuity conditions for the tangential components of the fields. One can notice that the time-harmonic [the time dependence is of the form $\exp(+j\omega t)$] field equations in the Veselago slab (region 2)

$$\nabla \times \mathbf{E} = j\omega\mu_0\mathbf{H}, \qquad \nabla \times \mathbf{H} = -j\omega\varepsilon_0\mathbf{E} \tag{1}$$

ISBN 978-952-67611-0-7

reg. 1	reg. 2	reg. 3
$\varepsilon = \varepsilon_0$	$\varepsilon = -\varepsilon_0$	$\varepsilon = \varepsilon_0$
$\mu = \mu_0$	$\mu = -\mu_0$	$\mu = \mu_0$
air	Veselago medium	air

$\longrightarrow z$

Fig. 1: An ideal Veselago lens: a planar slab of a backward-wave material with the medium parameters $\varepsilon = -\varepsilon_0$ and $\mu = -\mu_0$ in free space.

differ from the free-space equations outside the slab only by complex conjugation. A substitution

$$\mathbf{E}_{(\text{old})}, \mathbf{H}_{(\text{old})} \Rightarrow \mathbf{E}^*_{(\text{new})}, \mathbf{H}^*_{(\text{new})} \tag{2}$$

where into the field equations in region 2 results in a new but equivalent problem written for new field vectors. Here and thereafter * denotes the complex conjugation operation. In the new formulation the field equations are the same in all three regions, and they are simply the Maxwell equations in free space (1). The boundary conditions on the two interfaces, however, are no more the standard continuity conditions, but they involve complex conjugation:

$$\mathbf{E}_{\text{t}_{(1,3)}} = \mathbf{E}^*_{\text{t}_{(2)}}, \qquad \mathbf{H}_{\text{t}_{(1,3)}} = \mathbf{H}^*_{\text{t}_{(2)}} \tag{3}$$

We observe that the problem involving an ideal Veselago slab is mathematically equivalent to the problem dealing with a pair of conjugating surfaces in free space. Hence, for the latter system the field solutions are the same as for a Veselago slab, as well as the physical phenomena taking place at the interfaces: the propagating plane waves are refracted negatively at the interface, and the evanescent modes are resonantly enhanced. Thus, a pair of phase conjugating planes really makes a perfect lens.

Next we notice that the boundary conditions (3) imply non-continuity of the both tangential electric and magnetic fields across the phase conjugating plane. The jumps of the fields can be expressed as follows:

$$\mathbf{E}_\text{t}(x,y)\Big|_{z=+0} - \mathbf{E}_\text{t}(x,y)\Big|_{z=-0} = -2\,\text{Im}(\mathbf{E}_\text{t}(x,y))\Big|_{z=-0} \tag{4}$$

$$\mathbf{H}_\text{t}(x,y)\Big|_{z=+0} - \mathbf{H}_\text{t}(x,y)\Big|_{z=-0} = -2\,\text{Im}(\mathbf{H}_\text{t}(x,y))\Big|_{z=-0} \tag{5}$$

From the other hand, these jumps are related with the equivalent magnetic and electric surface currents that exist on the surface:

$$\mathbf{z}_0 \times \mathbf{J}_\text{m}(x,y) = \mathbf{E}_\text{t}(x,y)\Big|_{z=+0} - \mathbf{E}_\text{t}(x,y)\Big|_{z=-0} \tag{6}$$

$$-\mathbf{z}_0 \times \mathbf{J}_\text{e}(x,y) = \mathbf{H}_\text{t}(x,y)\Big|_{z=+0} - \mathbf{H}_\text{t}(x,y)\Big|_{z=-0} \tag{7}$$

Let us decompose each of these surface currents into a sum of two currents: $\mathbf{J}_\text{e} = \mathbf{J}_\text{e}^{(1)} + \mathbf{J}_\text{e}^{(2)}$, $\mathbf{J}_\text{m} = \mathbf{J}_\text{m}^{(1)} + \mathbf{J}_\text{m}^{(2)}$, where

$$-\mathbf{z}_0 \times \mathbf{J}_\text{e}^{(1)} = -\mathbf{H}_\text{t}\Big|_{z=-0}, \qquad \mathbf{z}_0 \times \mathbf{J}_\text{m}^{(1)} = -\mathbf{E}_\text{t}\Big|_{z=-0} \tag{8}$$

$$-\mathbf{z}_0 \times \mathbf{J}_\text{e}^{(2)} = \mathbf{H}_\text{t}\Big|_{z=+0}, \qquad \mathbf{z}_0 \times \mathbf{J}_\text{m}^{(2)} = \mathbf{E}_\text{t}\Big|_{z=+0} \tag{9}$$

One can see that the pair of the surface currents $\mathbf{J}_\text{e}^{(1)}$, $\mathbf{J}_\text{m}^{(1)}$ is essentially an equivalent Huygens source defined at the plane $z = -0$. In the half-space $z > 0$ this source produces the field which is the negative

ISBN 978-952-67611-0-7

of the field that the external sources located at $z < 0$ induce in the half-space $z > 0$ (the negation is due to the minus signs in the right-hand side of (8)). Thus, the physical role of the currents $\mathbf{J}_{\rm e}^{(1)}$, $\mathbf{J}_{\rm m}^{(1)}$ when concerned with the half-space $z > 0$ is to cancel the field incident from the other half-space. The same holds for the other pair of currents $\mathbf{J}_{\rm e}^{(2)}$, $\mathbf{J}_{\rm m}^{(2)}$ when concerned with the half-space $z < 0$. These currents form a Huygens source defined at $z = +0$ plane. In the half-space $z < 0$ they cancel the field produced by the external sources from the $z > 0$ half-space.

From the other hand, the pair of currents $\mathbf{J}_{\rm e}^{(1)}$, $\mathbf{J}_{\rm m}^{(1)}$ plays another role when concerned with the half-space $z < 0$. Indeed, using the boundary conditions (3) we write

$$-\mathbf{z}_0 \times \mathbf{J}_{\rm e}^{(1)} = -\mathbf{H}_{\rm t}^*\Big|_{z=+0}, \quad \mathbf{z}_0 \times \mathbf{J}_{\rm m}^{(1)} = -\mathbf{E}_{\rm t}^*\Big|_{z=+0} \qquad (10)$$

from which it is evident that these currents can be identified also as an equivalent source located at the plane $z = +0$ that produces at $z < 0$ the conjugated field of the sources located in the half-space $z > 0$. Respectively, $\mathbf{J}_{\rm e}^{(2)}$, $\mathbf{J}_{\rm m}^{(2)}$ produce the conjugated field in the region $z > 0$.

We can summarize that the ideal phase conjugating surfaces that realize perfect lens can be constructed as Huygens-source combinations of electric and magnetic surface currents, which are excited by the magnetic and electric fields, respectively, via non-linear phase conjugating elements. In the presentation we will present an analysis of a particular simple realization based on non-linearly loaded electric and magnetic dipoles and estimate the transformation efficiency.

3. Conclusion

In this presentation we will re-visit the idea of the perfect lens based on phase conjugating surfaces. In the original publication of 2003 [2], the formulation was based on ideal sheet conditions for tangential electric and magnetic fields, and approaches to possible realizations using non-linear materials and three-wave mixing were only briefly and conceptually mentioned. Here we show that the ideal complex conjugating sheet conditions are not only a mathematical abstraction, but these conditions are physically realizable with non-linear Huygens elements. We will outline possible designs of appropriate physical structures.

Acknowledgement

This work was supported in part by the Aalto University School of Electrical Engineering via a visiting professor grant and by the Finnish Academy, Aalto University, and Nokia via the Center of Excellence programme.

References

[1] J. Pendry, Phys. Rev. Lett. vol. 85, p. 3966, 2000.
[2] S. Maslovski and S. Tretyakov, Phase conjugation and perfect lensing, *Journal of Applied Physics*, vol. 94, no. 7, pp. 4241-4243, 2003.
[3] C.A. Allen, K.M.K.H. Leong, and T. Itoh, A negative reflective/refractive “meta-interface” using a bi-directional phase-conjugating array, *IEEE Int. Microw. Theory Techn. Symp. Digest*, vol. 3, pp. 1875-1878, 2003.
[4] J.B. Pendry, Time reversal and negative refraction, *Science*, vol. 322, pp. 71-73, 2008.
[5] V.F. Fusco, N.B. Buchanan, and O. Malyuskin, Active phase conjugating lens with sub-wavelength resolution capability, *IEEE Trans. Antennas Propag.*, vol. 58, pp. 798-808, 2010.
[6] A.R. Katko, S. Gu, J.P. Barrett, B. Popa, G. Shvets, and S.A. Cummer, Phase conjugation and negative refraction using nonleinear active metamaterials, *Phys. Rev. Lett.*, vol. 105, 123905, 2010.

