Asymmetric transmission in planar chiral metamaterials: microscopic explanation

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Abstract

Lorentz electron theory is a powerful approach for description of macroscopic parameters of a medium based on microscopic characteristics of the individual electron. For a planar array of chiral metallic split rings, we determine the averaged electron's characteristics in a sprit ring and apply them to derive the permittivity tensor of the homogenized medium. The effective material parameters obtained describe anisotropic dichroic material, where electromagnetic waves are governed by enantiomeric and directional asymmetry – the signature property of planar chiral metamaterials.

1. Introduction

Planar metamaterial (PMM) is a distinct type of metamaterials, because it possesses several unique properties different from those in a bulk metamaterial or Faraday media [1, 2]. One would expect that highly asymmetric 2D structure elements will be equivalent to 3D elements in the optical properties of the metamaterial, featuring "giant optical activity" as it happens in bulk optically active media (see, e.g., [3]). However, this is not the case. In PMMs, elliptical polarization eigenstates appear to be co-rotating in contrast to the counter-rotating eigenpolarizations of ordinary chiral or Fadaray media [1]. As a result, transmission spectra of PMMs change when the direction of incidence is reversed or when the original structure is replaced by its enantiomer (i.e., mirror image).

These peculiar optical properties of PMMs were recently shown to result from the fact that it is dichroism rather than gyrotropy which is responsible for the chiral properties in PMMs [4]. A microscopic-level theory explaining the origin of this dichroism is still missing. In this paper we discuss the Lorentz-theory averaging scheme for the split-rings. By applying conventional homogenization procedures, the effective permittivity tensor is derived and shown to be responsible for the optical properties observed in PMMs.

2. Dielectric permittivity tensor for the planar split-ring metamaterial

We consider the planar metamaterial where the element is a double-split metal ring with radius R and width d embedded into the dielectric matrix (see the inset in Fig. 1). The thickness of the metamaterial slab c_0 is comparable with the skin depth of the metal, but the substrate is thick ($D \gg c_0$). The double-split ring is asymmetric in the sense that it has no in-plane mirror symmetry axis. As such, there exist two distinct mirror images (enantiomers) of the same structure, which cannot be superimposed without lifting off the plane. This is totally different from the case of the single-split ring, which is always mirror-symmetric in the plane and has no distinct enantiomers. A monochromatic plane wave with angular frequency ω normally impinges on the structure.

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The proposed averaging scheme is based on the properties of the individual classical electron in the external fields. The dynamics of the electron is defined by the characteristics averaged over the cubic unit cell. The electron is affected by the driving force $e\mathbf{E}^m$, the friction force $-m\gamma\mathbf{v}$, and the screening force originating from the interaction of the electron with the charges induced at the edges of the metal (m, e, and $\mathbf{v}$ are the mass, charge, and velocity of the electron, γ is the effective decay frequency, and $\mathbf{E}^m$ is the averaged electric field in the metal.) The induced charges almost completely compensate the radial external force, so that the electrons can move only in azimuthal direction. The azimuthal screening force can be approximated by the Coulomb law, if the point charges are induced at the split ring tips defined by $\varphi = \beta_1/2$, $\beta_1/2 + \alpha_1$, $\beta_1/2 + \alpha_1 + \beta_2$, and $\beta_1/2 + \alpha_1 + \beta_2 + \alpha_2$.

The displacement of the electrons in the two ring segments can be found from coupled equations, with interaction between the segments accounted for. When the position of the electrons are solved for, we apply the conventional homogenization procedure (see [5] and references therein). We then retrieve the polarizability of a single ring, taking into account only electric dipole moments. Next-order contributions (magnetic dipole and electric quadrupole moments) can be omitted for normal incidence of the wave on a planar metamaterial [6]. The interaction between the neighboring split rings is included using the interaction matrix approach [7]. The resulting effective permittivity tensor can be presented in the form of a symmetric matrix

$$\varepsilon_{\text{eff}}(\omega) = \begin{pmatrix} \varepsilon_{11}(\omega) & \varepsilon_{12}(\omega) & 0 \\ \varepsilon_{12}(\omega) & \varepsilon_{22}(\omega) & 0 \\ 0 & 0 & \varepsilon_{33}(\omega) \end{pmatrix}. \tag{1}$$

This permittivity tensor with complex components describes anisotropic, elliptically dichroic media. The relevant components of this matrix are plotted in Fig. 1(a) and (b). It can be seen that the PMM in question exhibits a particle-plasmon resonant response as reported earlier [2, 4].

From the permittivity tensor in Eq. (1) one can determine the polarization states of the eigenwaves. They turn out to be elliptically polarized. The rotation direction for the first wave possessing magnetic field strength $\mathbf{H}_1$ can be extracted from $\eta_1 = \mathrm{i}\mathbf{e}_z(\mathbf{H}_1 \times \mathbf{H}_1^*)$, where $\mathbf{e}_z$ is the unit vector along the direction of the wave propagation. The wave is right-handed if $\eta_1 > 0$ and left-handed otherwise. A similar quantity $\eta_2 = \mathrm{i}\mathbf{e}_z(\mathbf{H}_2 \times \mathbf{H}_2^*)$ is determined for the second eigenwave. The parameter $\eta = \eta_1\eta_2$ indicates whether the eigenwaves have co-rotating (if $\eta > 0$) or counter-rotating (if $\eta < 0$) polarizations. Fig. 1(c) shows the calculated η in the frequency range around the resonance. It is seen that $\eta > 0$, so we conclude that the waves are co-rotating. Thus, we have shown microscopically that dichroism (rather than gyrotropy) plays the main part in explaining the effects arising in PMMs.

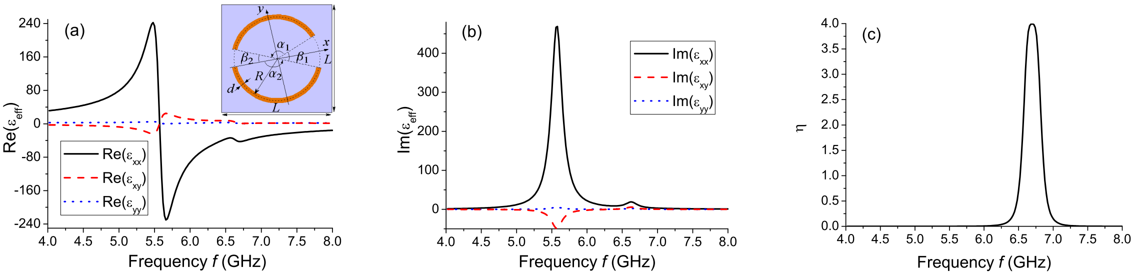

Fig. 1: (a) Real and (b) imaginary parts of the components of the effective dielectric permittivity tensor for the split-ring metamaterial. (c) Coefficient η vs. frequency. In the inset the split ring is shown. Dimensions of the split-ring cell $L = 15$mm, ring radius $R = 6$ mm, ring width $d = 0.8$ mm, ring thickness $c_0 = 35$ μm, substrate thickness $D = 1.6$ mm, $\alpha_1 = 7\pi/9$, $\alpha_2 = 8\pi/9$, $\beta_1 = 2\pi/9$, $\beta_2 = \pi/9$, substrate permittivity $\varepsilon_d = 2.25$.

3. Asymmetric transmission

Transmission spectra $T(\omega)$ for the metamaterial slab can be further computed using the transfer matrix approach for the known incident fields and permittivity tensors of the media. We consider two incident waves: right-handed circularly polarized (RCP) and left-handed circularly polarized (LCP) waves. The transmissions in both cases are shown in Fig. 2(a). The difference in transmission should disappear if the split rings possess a mirror-symmetry axis (when $\beta_1 = \beta_2$ or $\alpha_1 = \alpha_2$) because the structure no longer has two distinct enantiomers. Indeed, in Fig. 2(b) the transmission difference $\Delta T = T_{LCP} - T_{RCP}$ equals zero for the split-ring geometry with $\beta_1 = \beta_2 = 1.5\pi/9$. It is also seen that if the structure is replaced with its enantiomeric counterpart (i.e., if the angles β_1 and β_2 are exchanged), the transmission spectra for LCP and RCP are exchanged and $\Delta T(\omega)$ changes sign. As switching enantiomers is equivalent to reversing the direction of incidence for a circularly polarized wave, it is confirmed that directional asymmetry is manifest through asymmetry in the PMM's response to LCP vs. RCP incident light.

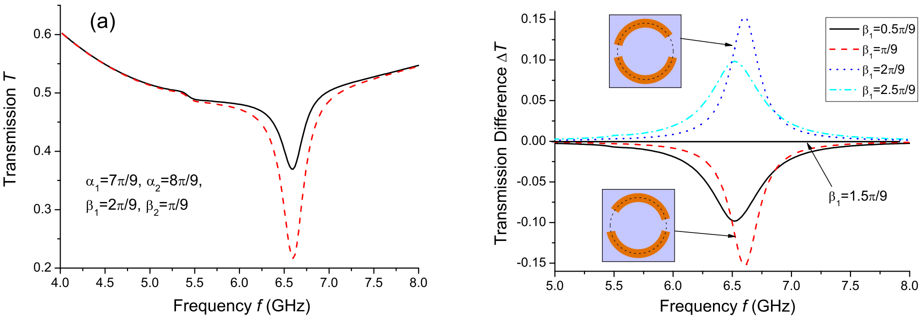

Fig. 2: (a) Transmission spectra of LCP (solid line) and RCP (dashed line) waves. (b) Difference of transmissions of LCP and RCP incident waves for varying β_1 and $\beta_2 = 3\pi/9 - \beta_1$.

4. Conclusion

An averaging procedure for effective microscopic parameters of an electron in planar metamaterials is proposed and demonstrated using planar chiral split-ring metamaterials as an example. We reveal that the effective parameters of the split-ring structure support co-rotating eigenpolarizations, signature to chiral PMMs [4]. The calculated optical spectra possess the directional asymmetry as previously reported [2].

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