Approaches to Transformation Optical Design in 3D

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Abstract

We present several approaches to realizing Transformation Optical designs in three dimensions. We employ quasi-conformal transformations and chiral inclusions to mitigate the task of creating these devices. We verify the performance of each design via numerical ray-tracing and finite element simulation of a flattened Luneburg lens and an electromagnetic cloak.

1. Introduction

Transformation Optics (TO) is a method whereby coordinate transformations between virtual and physical spaces are realized as electrically- and magnetically coupled- materials [1]. In recent years, TO has attracted a great deal of scientific interest due to the promise of unprecedented control of electromagnetic fields. Unfortunately, with few exceptions, the electromagnetic coupling required by TO is not provided by natural materials. To circumvent this limitation, researchers have turned to electromagnetic metamaterials (MMs) to approximate these complicated responses. Using MMs, researchers were able to demonstrate electromagnetic cloaking [2].

However, MMs pose significant design challenges for the would-be TO engineer. Prototypical MM unit cells [3,4] provide magnetic or electric responses in only one direction along a Cartesian axis. TO prescriptions, however, typically require distinct, non-zero coupling along spatially-varying principle axes. Moreover, the extreme material response of MMs is achieved by operating near resonance, which implies constraints on bandwidth and loss. Finally, the lattice parameter of most MMs is not much smaller than the wavelength of operation. This introduces artifacts in the response such as spatial dispersion and magneto-electric coupling [5-8].

Our goal is to ameliorate these complications by manipulating the material response and the coordinate transformations themselves. First, we show that the 2D concept of the quasi-conformal map (QCM) can be extended to 3D, and that the QCM provides several benefits in 3D. We then introduce the concept of transformation optics with chirality (CTO), and we explain how chiral materials may be used to ease the burden of design and fabrication.

2. Quasi-Conformal Mappings

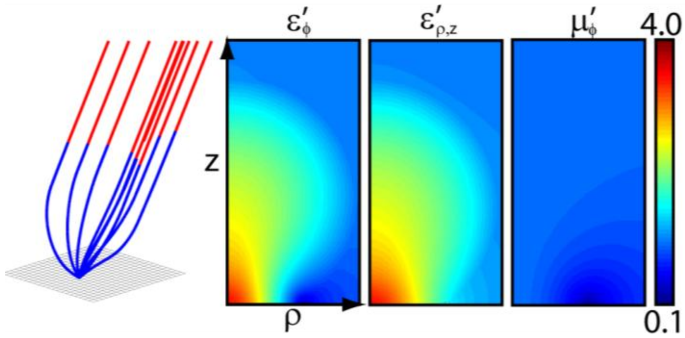

Fig. 1: (Left) ray-traces of a flattened Luneburg Lens. (Right) reduced material parameters for the lens.

Quasi-Conformal mappings (QCM) were introduced to TO by Li and Pendry to approximate 2D transformations with dielectric-only materials [9]. In this formulation, the transformation is invariant in one direction and therefore infinite in extent. However, if we perform a QCM in *cylindrical* coordinates, such that $\rho_p = \rho_p \;\; \rho_v, z_v$ and $z_p = z_p \;\; \rho_v, z_v$, then the transformation is bounded in space and invariant in the azimuthal direction, φ. The material parameters will take the form [10]:

$$\varepsilon_\varphi = \alpha/\beta \text{ and } \varepsilon_\rho \approx \varepsilon_z \approx \beta, \tag{1}$$

where α is the inverse of the Jacobian and $\beta = \rho_p/\rho_v$. The factor β appears because the metric in cylindrical coordinates is a function of the radial coordinates. The transformation must therefore compensate for this extra dilation in space in the material parameters. The responses in ρ and z are approximately equal for small deformations of the virtual domain.

The formulation in (1) may be simplified if we assume the device is electrically large. In this, the eikonal approximation, we can make the substitutions:

$$\varepsilon'_\varphi = \alpha, \varepsilon'_\rho = \varepsilon'_z = \beta^2, \mu'_\varphi = \alpha/\beta^2, \; \mu'_\rho = \mu'_z = 1, \tag{2}$$

and the performance of the device will be unchanged because we have preserved the anisotropic index of refraction. In this approximation, only one magnetic response is needed, which drastically reduces the complexity of design. We verify this approach via numerical ray-tracing of a flattened Luneburg Lens, which has already been demonstrated experimentally in 2D [11]. Ray traces and the requisite material parameters are shown in Fig. 1. We note that $\mu'_\varphi \leq 1$ everywhere, and could be realized

with metal plates [12]. These MMs have a very broadband response, and could allow for the device itself to perform well over a broad range of frequencies.

2. Transformation Optics with Chirality

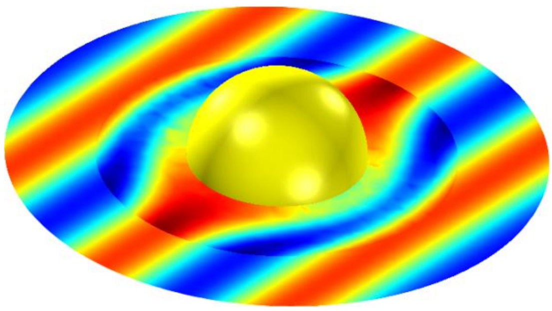

Fig. 2: Electric field (+ polarization) of a wave impinging on a copper sphere surrounded by a chiral cloak.

As we have shown, QCMs can be used to mitigate the material requirements in TO design. However, the QCM is restricted to deformations of quadrilaterals. Moreover, as the spatial warping increases, the approximation in (1) may no longer be valid. For more general TO designs, such as cloaks, alternatives must be found. One such alternative is to mimic the TO formulation with chirality (CTO).

The eikonal approximation allows us to define a local dispersion relation for TO media [13]:

$$\frac{k_1^2}{\gamma_2\gamma_3} + \frac{k_2^2}{\gamma_1\gamma_3} + \frac{k_3^2}{\gamma_2\gamma_3} = \frac{\omega^2}{c^2}, \tag{3}$$

where $\varepsilon = \mu = \gamma$ and the wave-vector components k_i are defined in terms of the eigenbasis of the material. If we could find a system with the same dispersion, we could mimic the performance of the device. For instance, the dispersion relation for anisotropic chiral medium with isotropic ε and μ is [14]:

$$\frac{k_1^2}{(1 \pm \kappa_2)(1 \pm \kappa_3)} + \frac{k_2^2}{(1 \pm \kappa_1)(1 \pm \kappa_3)} + \frac{k_3^2}{(1 \pm \kappa_1)(1 \pm \kappa_2)} = \varepsilon\mu \frac{\omega^2}{c^2} \tag{4}$$

where κ_i are the chirality parameters in each direction. The convention in (4) is that (+) refers to right-handed polarization and (-) to left-handed polarization. We can achieve the dispersion in (3) with (4) for a single handedness if we can find a medium such that $\kappa = (\varepsilon\mu)^{-1}\gamma \mp I$. For verification, we ran full-wave simulations of the canonical electromagnetic cloak [1] using this prescription for our chiral material. The results are shown in Fig 2. The performance is impressive considering that the device was designed in the eikonal limit, but the diameter is only two free-space wavelengths.

Chirality is beneficial to TO because it simplifies the MM unit cell design. As previously mentioned, TO calls for electric and magnetic responses in every direction. This would normally call for six separate metamaterial elements. However, a single chiral element can directly control the index of refraction in a given direction. Therefore, a maximum of three metamaterial elements would be required. Naturally, the use of chirality in this fashion restricts us to only one polarization of light. Additionally, chirality is necessarily dispersive, which will place constraints on bandwidth.

4. Conclusion

We have shown that complexity of TO devices may be drastically reduced through judicious choice of transformations and materials. We have demonstrated that both QCM and CTO can be used to reduce the number distinct material responses in a TO device from six to three. This reduction in complexity eases the burden in both design and fabrication. The QCM technique has the potential for broadband performance, but it is restricted in terms of the transformations that it can realize. The CTO approach, however, can be used to create more exotic devices over a narrow band.

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