Field-based transformation optics

A.V. Novitsky[1]

[1]DTU Fotonik, Department of Photonics Engineering, Technical University of Denmark, Ørsteds plads 343, DK-2800 Kgs. Lyngby, Denmark; e-mail: anov@fotonik.dtu.dk

Abstract

Instead of common definition of the transformation-optics devices via the coordinate transformation we offer the approach founded on boundary conditions for the fields. We demonstrate the effectiveness of the approach by two examples: two-shell cloak and concentrator of electric field. We believe that the field-based approach is quite important for effective field control.

1. Introduction. Electrodynamic cloak definition

Since the very beginning the transformation optics is assumed as the method of finding fields and material parameters for the known coordinate transformation [1, 2]. The latter defines the design of the device to be created: cloak, concentrator, rotator, etc. However the coordinate transformation itself is not necessary. It appears as the convenient intermediate characteristic which has no direct physical meaning. Here we present the transformation optics, from which the coordinate transformation is eliminated.

Let an incident wave is the plane wave propagating in z direction. What we know are the incident fields $\mathbf{E}' = \exp(\mathrm{i}k_0 z)\mathbf{e}_x$ and $\mathbf{H}' = \exp(\mathrm{i}k_0 z)\mathbf{e}_y$, material parameters of the ambient medium $\varepsilon' = \mu' = 1$ (we specify them as those for vacuum), and the fields in the cloaking shell $\mathbf{E}$ and $\mathbf{H}$. The goal is to determine the material parameters of transformation medium ε and μ.

Instead of geometrical definition of the transformation-optics device as coordinate transformation from the virtual to physical space, we use the so called electrodynamic definition. For example, for the cloak it is the set of boundary conditions leading to the invisibility : the cloaking shell should not scatter light and should not conduct light to the cavity under the shell:

$$\mathbf{E}_t(S_1) = \mathbf{E}'_t(S_1), \qquad \mathbf{H}_t(S_1) = \mathbf{H}'_t(S_1), \qquad \frac{c}{8\pi}\mathrm{Re}[\mathbf{E}(S_2) \times \mathbf{H}^*(S_2)]_n = 0, \tag{1}$$

where subscripts ”t” and ”n” denote tangential and normal components of the vectors (with respect to the outer S_1 and inner S_2 surfaces).

2. Material parameters

At first we solve the equations of transformation optics

$$\mathbf{E}(\mathbf{r}) = J^{-1}(\mathbf{r})\mathbf{E}'(\mathbf{r}), \qquad \mathbf{H}(\mathbf{r}) = J^{-1}(\mathbf{r})\mathbf{H}'(\mathbf{r}) \tag{2}$$

with respect to the Jacobian matrix J. The solution can be presented as [3]

$$J^{-1}(\mathbf{r}) = \mathbf{E} \otimes \frac{\mathbf{H}' \times \mathbf{P}'}{\mathbf{P}'^2} + \mathbf{H} \otimes \frac{\mathbf{P}' \times \mathbf{E}'}{\mathbf{P}'^2} + \mathbf{a} \otimes \mathbf{P}', \tag{3}$$

ISBN 978-952-67611-0-7

where $\mathbf{a} \otimes \mathbf{P}'$ is a dyad, $\mathbf{a}$ is an arbitrary vector, and $\mathbf{P}' = \mathbf{E}' \times \mathbf{H}'$. For the incident plane wave, we write

$$J^{-1} = \mathbf{E}_0 \otimes \mathbf{e}_x + \mathbf{H}_0 \otimes \mathbf{e}_y + \mathbf{a}_0 \otimes \mathbf{e}_z, \tag{4}$$

where $\mathbf{E}_0 = \mathrm{e}^{-\mathrm{i}k_0 z}\mathbf{E}$, $\mathbf{H}_0 = \mathrm{e}^{-\mathrm{i}k_0 z}\mathbf{H}$, and $\mathbf{a}_0 = \mathrm{e}^{2\mathrm{i}k_0 z}\mathbf{a}$. Jacobian matrix J has the special form and should satisfy equation

$$\nabla \times J^{-1}(\mathbf{r}) = 0. \tag{5}$$

Since vectors $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are constant, Eq. (5) straightforwardly leads us to the triple of conditions

$$\nabla \times \mathbf{E}_0 = 0, \qquad \nabla \times \mathbf{H}_0 = 0, \qquad \nabla \times \mathbf{a}_0 = 0 \tag{6}$$

or, via the scalar potentials ψ, χ, and η, to

$$\mathbf{E}_0 = \nabla\psi, \qquad \mathbf{H}_0 = \nabla\chi, \qquad \mathbf{a}_0 = \nabla\eta. \tag{7}$$

In terms of the common transformation optics, potentials have the meaning of the coordinates in the electromagnetic space, i.e. $\mathbf{r}' = \psi\mathbf{e}_x + \chi\mathbf{e}_y + \eta\mathbf{e}_z$. They completely define the fields as

$$\mathbf{E} = \mathrm{e}^{\mathrm{i}k_0\eta(\mathbf{r})}\nabla\psi(\mathbf{r}), \qquad \mathbf{H} = \mathrm{e}^{\mathrm{i}k_0\eta(\mathbf{r})}\nabla\chi(\mathbf{r}), \tag{8}$$

where we have replaced z with η in the cloak [3]. It is evident that ψ, χ, and η have the meaning of electric, magnetic, and phase potentials. Then the dielectric permittivity and magnetic permeability tensors read

$$\varepsilon = \mu = \frac{J^T J}{\det(J)} = \frac{\mathbf{a}_1 \otimes \mathbf{a}_1 + \mathbf{a}_2 \otimes \mathbf{a}_2 + \mathbf{a}_3 \otimes \mathbf{a}_3}{\nabla\psi(\nabla\eta \times \nabla\chi)}, \tag{9}$$

where $\mathbf{a}_1 = (\nabla\chi \times \nabla\eta)$, $\mathbf{a}_2 = (\nabla\psi \times \nabla\chi)$, and $\mathbf{a}_3 = (\nabla\eta \times \nabla\psi)$. The first two boundary conditions in Eq. (1) take the form

$$\eta(S_1) = z(S_1), \qquad \nabla_t\psi(S_1) = \mathbf{E}'_t(S_1), \qquad \nabla_t\chi(S_1) = \mathbf{H}'_t(S_1). \tag{10}$$

The first expression holds for the phase at the boundary S_1. To obtain the general solutions, we have to solve the differential equations. However, sometimes the particular solutions can be written as the Cartesian coordinates in terms of the curvilinear coordinates. Then we have $\psi(S_1) = x(b, x_2, x_3)$, $\chi(S_1) = y(b, x_2, x_3)$, $\eta(S_1) = z(b, x_2, x_3)$ where coordinates x_2 and x_3 describe the interface S_1, x_1 is orthogonal to the interface, $x_1 = a$ and $x_1 = b$ describe the inner and outer cloak boundaries. We choose the potentials in straightforward manner as $\psi(\mathbf{r}) = f(x_1)x(b, x_2, x_3)$, $\chi(\mathbf{r}) = g(x_1)y(b, x_2, x_3)$, and $\eta(\mathbf{r}) = h(x_1)z(b, x_2, x_3)$. The boundary condition at S_2 takes the form $f(a)g(a)(\nabla_t x \times \nabla_t y)_n = 0$, which is equivalent to $f(a) = 0$ or $g(a) = 0$. Finally, the boundary conditions can be rewritten as

$$\begin{gathered} f(b) = 1, \qquad g(b) = 1, \qquad h(b) = 1, \\ f(a) = 0 \qquad \text{or} \qquad g(a) = 0. \end{gathered} \tag{11}$$

3. Discussion and examples

We have considered the particular wave incident onto the transformation medium. The solution for such a wave allows us to determine the potentials ψ, χ, and η. Now, for the known potentials we have the material tensors and Jacobian matrix. If the incident wave is arbitrary, from Eq. (2) we get

$$\mathbf{E}(\mathbf{r}) = (\nabla\psi \otimes \mathbf{e}_x + \nabla\chi \otimes \mathbf{e}_y + \nabla\eta \otimes \mathbf{e}_z)\mathbf{E}'(\mathbf{r}). \tag{12}$$

The main problem is to find the potentials for some special case. Then any incident field can be described.

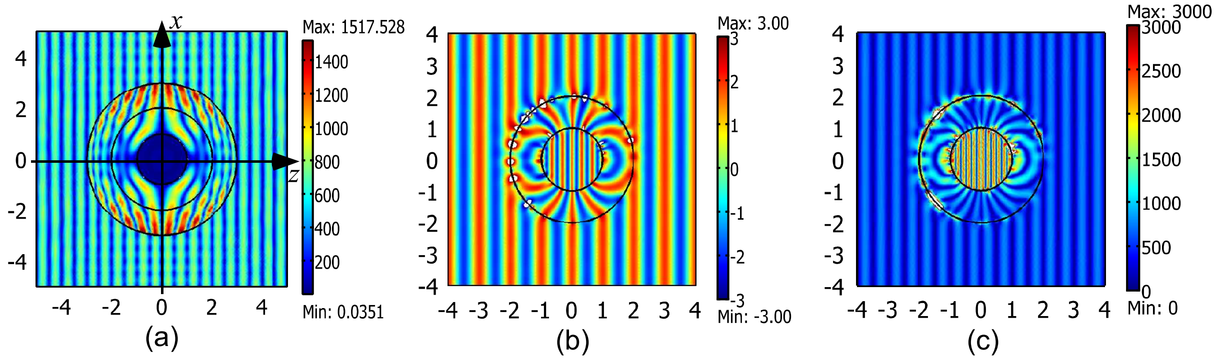

Fig. 1: (a) Electric field $|\mathbf{E}|$ in two-shell cloak. In concentrator, (b) magnetic field H_z is not enhanced, while (c) electric field $|\mathbf{E}|$ is three-times amplified.

For the cylindrical cloak the potentials can be represented in the form [3]

$$\psi = f(r)\cos\varphi, \qquad \chi = g(r)y, \qquad \eta = h(r)\sin\varphi. \tag{13}$$

Further we will take the special case $f(r) = h(r)$ and $g(r) = 1$. Then the boundary conditions are met when $f(b) = b$ and $f(a) = 0$. If there are several transformation media, the field in each of them is described via the potentials. At the interfaces between the media, the tangential components of the fields are continuous. For example, let the cloak consists of two transformation media, the first occupies the cylindrical layer $b \geq r \geq c$, the second does $c > r \geq a$. In the first medium, we can take $f_1(r) = r^2/b$, which provides the absence of scattering at $r = b$. In the second medium, the function should satisfy two conditions, $f_2(c) = f_1(c) = c^2/b$ and $f_2(a) = 0$, e.g. as $f_2(r) = (c^2/b)(r-a)/(c-a)$. Such a two-shell cloak is demonstrated in Fig. 1(a) (COMSOL Multiphysics is used for simulations). Note that none of the shells is a cloak, but they operate as invisibility cloak together.

To form a concentrator [4], we need to satisfy the no-scattering boundary condition at the outer interface ($f(b) = b$) and link the transformation-medium solution with the plane wave field in the cavity. We can amplify only electric field, if choose appropriately the material parameters in the cavity. For example, in Fig. 1(b) and (c) we use $f(r) = b(r - a + (b-r)Aa/b)/(b-a)$, where A is the amplitude of the electric field inside the cavity (amplification).

In conclusion, the devices of transformation optics can be defined in terms of the usual electrodynamics. We have demonstrated this assertion by two examples: invisibility cloak and concentrator. It should be noted that in principle any field agreed with Maxwell equations can be generated in the cavity. Then transformation medium making an arbitrary beam from the incident plane wave can be considered as a metamaterial lens.

Financial support from the Danish Research Council for Technology and Production Sciences via project THz COW is acknowledged.

References

[1] J.B. Pendry, D. Schurig, and D.R. Smith, Controlling electromagnetic fields, *Science*, vol. 312, p. 1780–1783, 2006.

[2] U. Leonhardt, Optical conformal mapping, *Science*, vol. 312, p. 1777–1780, 2006.

[3] A.V. Novitsky, Inverse problem in transformation optics, *J. Opt.*, vol. 13, 035104, 2011.

[4] M. Rahm, D. Schurig, D.A. Roberts, S.A. Cummer, D.R. Smith and J.B. Pendry, Design of electromagnetic cloaks and concentrators using form-invariant coordinate transformations of Maxwell's equations, *Photon. Nanostruct.: Fundam. Appl.*, vol. 6, p. 87–95, 2008.

