Tensor Circuit Networks for Transformation Optics

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Abstract

In this presentation, we will describe transmission-line based metamaterials that can possess tensor constitutive parameters. The utility of tensor analysis in the design of such metamaterials will be shown. In addition, various microwave devices implemented using tensor transmission-line metamaterials will be demonstrated.

1. Introduction

Transmission-line (TL) metamaterials possessing effective material parameters that are diagonal have been studied for some time [1,2]. Recently, transmission-line based metamaterials that can exhibit tensor constitutive parameters were introduced by the authors [3,4]. Through the tensor analysis of electrical networks, it was shown that material parameter distributions of transformation-designed electromagnetic devices can be directly mapped to two-dimensional transmission-line networks. The work drew a connection between microwave network theory and transformation optics[5], bringing to light "transformation circuits". Since these tensor metamaterials are TL based (travelling-wave structures), they promise broad bandwidths of operation and low losses. They allow extreme control of electromagnetic fields along a surface / radiating aperture, and therefore will find broad application and enable novel microwave devices and antennas designed through transformation optics.

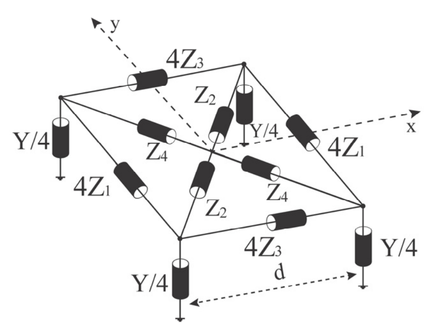

Fig. 1: An unit cell of a tensor transmission-line metamaterial (perspective view)

In the homogeneous (long wavelength) limit, tensor TL metamaterials can be represented by an impedance tensor and an admittance [3]. A one-to-one relationship can then be drawn between the impedance tensor and the metamaterial's effective permeability tensor, as well as the scalar admittance and its effective permittivity constant. For example, the 2D circuit network shown in Figure 1 can be represented by the admittance Y and the following impedance tensor

$$\overline{\overline{Z}} = \begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{bmatrix} \dfrac{2Z_3(Z_1Z_2 + Z_1Z_4 + Z_2Z_4)}{Z_D} & \dfrac{2Z_1Z_3(Z_2 - Z_4)}{Z_D} \\ \dfrac{2Z_1Z_3(Z_2 - Z_4)}{Z_D} & \dfrac{2Z_1(Z_2Z_3 + Z_2Z_4 + Z_3Z_4)}{Z_D} \end{bmatrix}, \quad (1)$$

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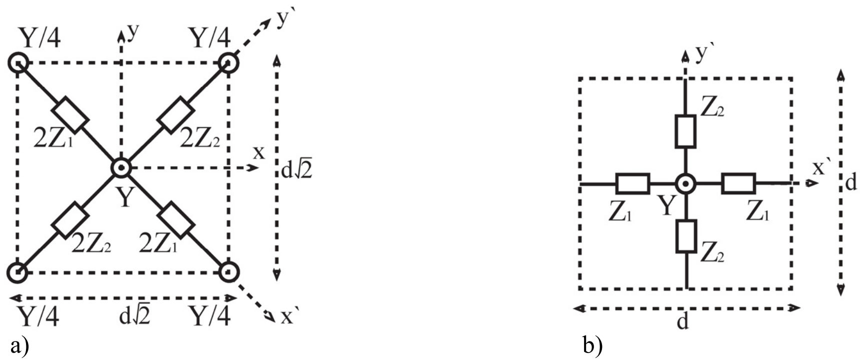

Fig. 2: Different unit cells for two rotated versions of the same transmission-line metamaterial (top view).

where $Z_D = Z_1Z_2 + 4Z_1Z_3 + Z_1Z_4 + Z_2Z_3 + Z_2Z_4 + Z_3Z_4$. The impedance tensor is found by removing the shunt admittance and applying voltages across the unit cell in the x and y directions and solving for the net currents [3].

If the series impedances (Z_1, Z_2, Z_3, Z_4) are reciprocal, the resulting impedance tensor will be symmetric ($z_{xy} = z_{yx}$), with only three distinct tensor entries. As a result, it is sufficient to use only three of the four series impedances to realize an arbitrary 2X2 impedance tensor. The remaining (fourth) impedance can be removed (set to infinity). Using certain impedances over others may result in simpler implementations and more suitable frequency dispersion of the material parameters.

For added design flexibility, we may also rotate the unit cell to achieve a desired impedance tensor. The additional degree of freedom provided by the rotation angle allows one to use only two of the four series impedances (Z_1, Z_2, Z_3, Z_4) to realize an arbitrary 2X2 impedance tensor. A rotation angle can always be found since the impedance tensor is symmetric, and therefore its eigenvectors form an orthonormal basis. The rotated impedance tensor can be expressed as

$$\overline{\overline{Z}}_r = A\overline{\overline{Z}}A^T = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix}\begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} , \qquad (2)$$

where A is the rotation matrix, and ϕ is the rotation angle from the x axis. As an example, let us consider the unit cells shown in Figure 2. Using the analysis presented in [3], we can find the impedance tensor of the unit cell shown in Figure 2a:

$$\overline{\overline{Z}} = \begin{bmatrix} Z_1 + Z_2 & Z_2 - Z_1 \\ Z_2 - Z_1 & Z_1 + Z_{21} \end{bmatrix} . \qquad (3)$$

If it is diagonalized, the tensor becomes

$$\overline{\overline{Z}} = \begin{bmatrix} 2Z_1 & 0 \\ 0 & 2Z_2 \end{bmatrix} \qquad (4)$$

along the principal l axes: $x' = 1/\sqrt{2}\hat{x} - 1/\sqrt{2}\hat{y}$ and $y' = 1/\sqrt{2}\hat{x} + 1/\sqrt{2}\hat{y}$. Therefore, the change of basis amounts to a rotation of 45 degrees. The diagonal Z tensor given by (4) can be written by inspection from Figure 2b, which shows a redrawn unit cell for the rotated metamaterial. The problem with utilizing rotation to achieve a given impedance tensor is the crystal misalignment that can arise at the interface between two tensor TL metamaterials rotated with respect to each other. In other words, unit cells rotated by different angles cannot easily be tiled together. Misalignment issues, however, do not arise between two different tensor TL unit cells implemented using circuit elements alone (three series impedances and no rotation of the unit cell).

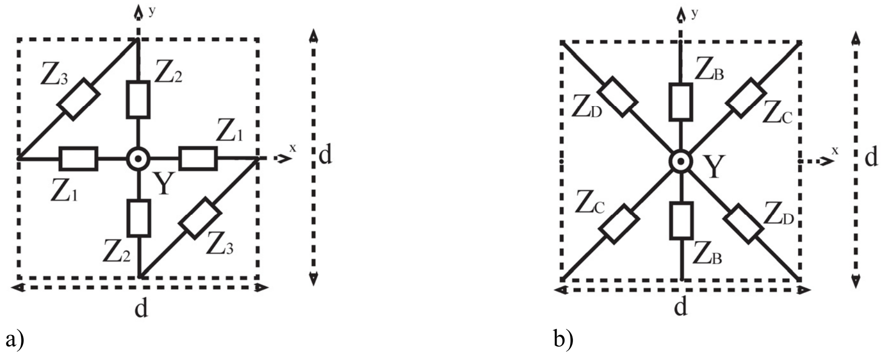

Fig. 3: Two different unit cell topologies for tensor TL metamaterials (top view).

The tensor analysis of circuit networks also allows us to draw equivalences between various metamaterial unit cell topologies. Let's consider the tensor TL metamaterial unit cells shown in Figure 3. The unit cell in Figure 3a has a shunt admittance Y and an impedance tensor given by the following expression

$$\overline{\overline{Z}} = \begin{bmatrix} \dfrac{2Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3} & \dfrac{-2Z_1Z_2}{Z_1+Z_2+Z_3} \\ \dfrac{-2Z_1Z_2}{Z_1+Z_2+Z_3} & \dfrac{2Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3} \end{bmatrix} . \quad (5)$$

The unit cell in Figure 3b also has a shunt admittance Y, but the following impedance tensor

$$\overline{\overline{Z}} = \begin{bmatrix} \dfrac{2(Z_BZ_C+Z_BZ_D+Z_CZ_D)}{4Z_B+Z_C+Z_D} & \dfrac{2Z_B(Z_C-Z_D)}{4Z_B+Z_C+Z_D} \\ \dfrac{2Z_B(Z_C-Z_D)}{4Z_B+Z_C+Z_D} & \dfrac{2Z_B(Z_C+Z_D)}{4Z_B+Z_C+Z_D} \end{bmatrix} . \quad (6)$$

By equating the tensor elements of the two unit cells, the series impedances (Z_n) of both structures can be chosen to produce metamaterials with identical propagation characteristics at a given operating frequency in the long wavelength limit.

At the conference, the presented tensor analysis will be elaborated upon. In addition, various tensor TL metamaterial topologies will be reported, and a few microwave devices designed using transformation optics and implemented using these tensor TL metamaterials will be shown.

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