Realization of D'B' boundary in terms of metamaterial

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Abstract

In this paper we aim to find a realization for the D'B' boundary conditions requiring vanishing of the normal derivatives of the normal components of the **D** and **B** fields. Since the realization of the DB boundary requiring vanishing of the normal components of the **D** and **B** fields is known, it is shown that the realization of the D'B' boundary can be based on a layer of suitable metamaterial which makes the transformation from DB to D'B' boundary.

1. Introduction

Electromagnetic boundary-value problems are normally defined in terms of impedance-boundary conditions involving linear relations between electric and magnetic field components tangential to the boundary surface. Denoting by **n** the unit vector normal to the boundary, the general form for such conditions can be written as

$$\mathbf{n}\times\mathbf{E} = \overline{\overline{\mathsf{Z}}}_s\cdot\mathbf{H}, \quad \text{or} \quad \mathbf{n}\times\mathbf{H} = \overline{\overline{\mathsf{Y}}}_s\cdot\mathbf{E} = 0, \tag{1}$$

where $\overline{\overline{\mathsf{Z}}}_s$ and $\overline{\overline{\mathsf{Y}}}_s$ are the respective surface-impedance and surface-admittance dyadics satisfying $\mathbf{n}\cdot\overline{\overline{\mathsf{Z}}}_s = \overline{\overline{\mathsf{Z}}}_s\cdot\mathbf{n} = 0$ and $\mathbf{n}\cdot\overline{\overline{\mathsf{Y}}}_s = \overline{\overline{\mathsf{Y}}}_s\cdot\mathbf{n} = 0$. Thus, normal components of the **E** and **H** vectors do not play any role in these boundary conditions. Typical examples of impedance-boundary conditions are the PEC and PMC conditions, $\mathbf{n}\times\mathbf{E} = 0$ and $\mathbf{n}\times\mathbf{H} = 0$, corresponding to the respective cases $\overline{\overline{\mathsf{Z}}}_s = 0$ and $\overline{\overline{\mathsf{Y}}}_s = 0$. It is, however, also possible to define boundary conditions in terms of normal components of the field vectors. In fact, conditions of the form

$$\mathbf{n}\cdot\mathbf{D} = 0, \quad \text{and} \quad \mathbf{n}\cdot\mathbf{B} = 0, \tag{2}$$

originally introduced in [1], have been shown to yield unique solutions to boundary-value problems [2]. Recently, the conditions (2) have proven to have importance in constructing electromagnetic cloaking structures [3, 4]. The conditions (2) have subsequently been dubbed as DB-boundary conditions [5, 6]. Other boundary conditions involving normal components and/or normal derivatives of normal components of the fields were introduced in [7] of which the set

$$\nabla\cdot(\mathbf{nn}\cdot\mathbf{D}) = 0, \quad \text{and} \quad \nabla\cdot(\mathbf{nn}\cdot\mathbf{B}) = 0 \tag{3}$$

ISBN 978-952-67611-0-7

was dubbed as D'B'-boundary conditions. It has been shown that objects with certain symmetry properties and defined by either DB or D'B' boundary conditions have zero backscattering, i.e., they cannot be seen by a radar [8].

All boundary conditions defined above are mathematical concepts. From the practical point of view there is the problem to realize them in terms of physical structures as closely as possible. It is known that the PEC boundary corresponds to an interface of an ideally conducting material which can be approximated by metals. In [1] it was shown that the DB boundary can be realized by an interface of an anisotropic medium whose normal permittivity and permeability parameters become zero, after which also other possibilities have been suggested. Up till now it has been a problem to find a realization for the D'B' boundary. The problem will be addressed in this paper by assuming a planar boundary $z = 0$, at which the D'B' conditions (3) have the form

$$\partial_z D_z = 0, \quad \partial_z B_z = 0. \tag{4}$$

The problem is handled in terms of a plane wave. It is obvious that, if the D'B' conditions are satisfied by an arbitrary plane wave, being linear, the conditions are satisfied by a field consisting of a sum or an integral of plane waves, in fact, by any electromagnetic field outside its sources.

As a guiding principle in the realization we use the knowledge that the eigenpolarizations for the plane wave reflecting from the D'B' plane, as well as from the DB plane, are the TE and TM polarizations with respect to the normal of the plane. In fact, one can show that an incident TE wave is reflected as a TE wave and an incident TM wave is reflected as a TM wave. Since it is known that the eigenwaves in a uniaxially anisotropic medium are also TE and TM polarized, it appears natural to try to find a realization in terms of such a medium.

2. Plane-wave in uniaxial anisotropic medium

Let us consider the uniaxial anisotropic medium defined by the medium equations

$$\mathbf{D} = \epsilon_t \mathbf{E}_t + \epsilon_z \mathbf{u}_z E_z, \quad \mathbf{B} = \mu_t \mathbf{H}_t + \mu_z \mathbf{u}_z H_z, \tag{5}$$

where $\mathbf{E}_t$ and $\mathbf{H}_t$ are components transverse to the z axis. Inserting the plane-wave expressions

$$\mathbf{E}(\mathbf{r}) = \mathbf{E} e^{-jk_x x} e^{-j\beta z}, \quad \mathbf{H}(\mathbf{r}) = \mathbf{H} e^{-jk_x x} e^{-j\beta z} \tag{6}$$

in the Maxwell equations and eliminating the transverse field components, two equations remain, one for $B_z = \mu_z H_z$ characterizing the TE fields, and the other one for $D_z = \epsilon_z E_z$ characterizing the TM fields,

$$(\beta_{TE}^2 + k_x^2(\mu_t/\mu_z) - k_t^2)B_z = 0, \quad (\beta_{TM}^2 + k_x^2(\epsilon_t/\epsilon_z) - k_t^2)D_z = 0, \tag{7}$$

with $k_t = \omega\sqrt{\mu_t \epsilon_t}$. The propagation factors can be solved as

$$\beta_{TE} = \sqrt{k_t^2 - k_x^2(\mu_t/\mu_z)}, \quad \beta_{TM} = \sqrt{k_t^2 - k_x^2(\epsilon_t/\epsilon_z)}. \tag{8}$$

3. Layer of uniaxial medium

Let us consider a layer of uniaxially anisotropic medium extending from $z = 0$ to $z = d$ and assume suitable boundary conditions at $z = d$. The field components in the layer consist of forward and backward waves as

$$D_z(x, z) = e^{-jk_x x}(D_+ e^{-j\beta_{TM} z} + D_- e^{j\beta_{TM} z}), \tag{9}$$

$$B_z(x, z) = e^{-jk_x x}(B_+ e^{-j\beta_{TE} z} + B_- e^{j\beta_{TE} z}). \tag{10}$$

ISBN 978-952-67611-0-7

For the realization of the D'B' conditions at the interface $z = 0$ we require

$$\partial_z D_z(x,0) = -j\beta_{TM} e^{-jk_x x}(D_+ - D_-) = 0, \tag{11}$$

$$\partial_z B_z(x,0) = -j\beta_{TE} e^{-jk_x x}(B_+ - B_-) = 0, \tag{12}$$

which implies $D_- = D_+$ and $B_+ = B_-$. To achieve this, we ask what boundary conditions must be imposed at $z = d$ where the fields are

$$D_z(x,d) = 2e^{-jk_x x} D_+ \cos\beta_{TM} d, \tag{13}$$

$$B_z(x,d) = 2e^{-jk_x x} B_+ \cos\beta_{TE} d. \tag{14}$$

It now appears possible to obtain the D'B' conditions at $z = 0$ in terms of DB conditions at $z = d$. Requiring $D_z(x,d) = 0$ and $B_z(x,d) = 0$ we must simultaneously have $d = \pi/2\beta_{TM}$ and $d = \pi/2\beta_{TE}$ for all possible values of k_x. Considering the form of the expressions (8), it is obvious that this can be achieved by requiring

$$\epsilon_z/\epsilon_t \to \infty, \qquad \mu_z/\mu_t \to \infty. \tag{15}$$

Because in this case we have $\beta_{TE} = \beta_{TM} = k_t$, we can choose $d = \pi/2k_t$ and the case appears to be solved. A material involving infinite parameters μ_z, ϵ_z has been previously called by the name wave-guiding medium [9].

One may note that the thickness d of the layer can be made as small as wished by letting $\mu_t\epsilon_t$ grow large enough. Here we must, however, separate the two grades of largeness involved in the axial and transverse medium quantities. However, recalling that the realization of the DB boundary by assuming vanishing axial parameters may also lead to a thin sheet of material, the final realization of the D'B' boundary may be achieved theoretically by a thin double sheet. Because the boundary conditions are local, the same thin sheet realization apparently remains valid for curved boundary surfaces as well.

4. Conclusion

A realization for the planar D'B' boundary has been found in terms of a layer of metamaterial above the DB boundary, whose realization has been given before. Thus, the open question whether the D'B' boundary is just a mathematical artefact without a physical counterpart has been answered.

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