

# Negative refraction by a two-sided mushroom structure with loaded vias

C. S. R. Kaipa<sup>1</sup>, A. B. Yakovlev<sup>1</sup>, M. G. Silveirinha<sup>2</sup>, and S. I. Maslovski<sup>2</sup>

<sup>1</sup>Department of Electrical Engineering, University of Mississippi, University, MS 38677-1848 USA, email: ckaipa@olemiss.edu, yakovlev@olemiss.edu

<sup>2</sup>Departamento de Engenharia Electrotécnica, Instituto de Telecomunicações, Universidade de Coimbra Pólo II, 3030-290 Coimbra, Portugal, email: mario.silveirinha@co.it.pt, stas@co.it.pt

## Abstract

In this paper we show that the two-sided mushroom structure with inductive loadings (as lumped loads) at the junction of vias and metallic patches exhibits negative refraction. The transmission properties are analysed based on the nonlocal homogenization model for the mushroom structure with a generalized additional boundary condition for loaded vias. It is shown that it is possible to control the negative refraction angle by varying the inductive loads.

## 1. Introduction

Recently, bulk material formed by periodically attaching metallic patches to wire medium (WM) [1], has been shown to exhibit negative refraction at microwaves, with suppressed spatial dispersion (SD) effects as originally suggested in [2]. Building on this idea, in [3] it is shown that by inclusion of the air gap in between the metamaterial layers it is possible to control the negative refraction angle. In this work we consider the two-sided mushroom structure formed with loaded vias (impedance insertions at the via-to-patch connection) and demonstrate that the structure exhibits strong negative refraction. We investigate the effect of varying the load (inductive), and show that this configuration enables the control of negative refraction angle. The analysis is carried out using the nonlocal homogenization model with the generalized additional boundary condition (GABC) derived in [4] at the junction with impedance loadings.

## 2. Two-sided mushroom structure

The geometry of the two-sided mushroom structure with loaded vias is shown in Fig. 1 with the TM-polarized plane-wave incidence. The patch arrays are at the planes  $z = 0$  and  $z = -h$  and the vias are connected to the metallic patches through lumped loads. The homogenization model for the analysis of reflection/transmission properties of the mushroom structure is based on the spatially-dispersive model for the wire media ([5] and references therein) with the GABC's for the microscopic current  $I(z)$  at the connection of lumped loads to metallic patches [4]:

$$\left[ \frac{dI(z)}{dz} + \left( j\omega C Z_{\text{Load1}} + \frac{C}{C_{\text{patch}}} \right) I(z) \right]_{z=0} = 0 \quad (1)$$

$$\left[ \frac{dI(z)}{dz} - \left( j\omega C Z_{\text{Load2}} + \frac{C}{C_{\text{patch}}} \right) I(z) \right]_{z=-h} = 0, \quad (2)$$

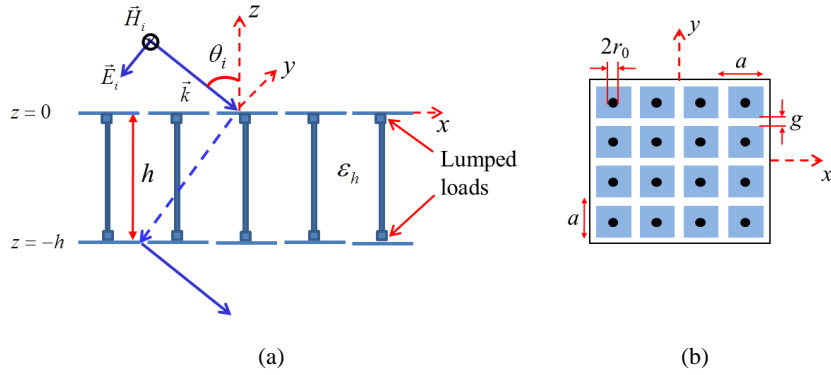


Fig. 1: Geometry of the two-sided mushroom structure with loaded vias illuminated by an obliquely incident TM-polarized plane wave: (a) Cross-section view and (b) top view.

where  $C = 2\pi\epsilon_0\epsilon_h / \log(a^2/4r_0(a-r_0))$  is the capacitance per unit length of the wire medium,  $C_{\text{patch}} = \pi\epsilon_0(\epsilon_h + 1)(a-g) / \log(\sec \frac{\pi g}{2a})$  is the capacitance of the patch in a regular array of patches given in [6], and  $Z_{\text{Load}1,2}$  are the impedances of lumped loads. We may neglect the term  $C/C_{\text{patch}}$  in the GABC's Eqs. 1 and 2 when the gap ( $g$ ) is small and  $C_{\text{patch}} \gg hC$ . Since the insertion of loads in the wire introduces non-uniformity in the current and charge distributions, the correction terms described in terms of the parasitic capacitance  $C_{\text{par}}$  and parasitic inductance  $L_{\text{par}}$  should be taken into account in the above equations for load impedances [4],

$$Z_{\text{Load,eff}1,2} = j\omega L_{\text{par}} + \frac{1}{j\omega C_{\text{par}} + (1/Z_{\text{Load}1,2})}. \quad (3)$$

The reflection/transmission properties can now be obtained by matching the tangential components of the electric and magnetic fields at the air-patch interfaces by using the two-sided impedance boundary conditions, and the above GABC's Eqs. 1 and 2 at the connection of loaded vias to the metallic patches.

### 3. Results and discussion

Let us consider the case of the two-sided mushroom structure with the vias connected to the metallic patches through lumped loads at the plane  $z = -h$  and a true short circuit (SC) at the plane  $z = 0$ . The dimensions of the structure (with the notations as shown in Fig. 1) are as follows:  $\theta_i = 60^\circ$ ,  $\epsilon_h = 10.2$ ,  $a = 2$  mm,  $g = 0.2$  mm,  $r_0 = 0.05$  mm, and  $h = 2$  mm. Fig. 2 (a) demonstrates the transmission magnitude and phase for the two-sided mushroom structure with an inductive load of 0.2 nH as a function of frequency. The inductive load is connected to the patch through a gap of 0.1 mm. It can be seen that the homogenization results are in a reasonable agreement with the HFSS full-wave results. It is observed that with an increase in the inductive load, there is a decrease in the transmission resonance (corresponding to the maximum transmission) and in the plasma frequency (plasma frequency  $f_p \approx 12.14$  GHz for SC case). In order to study the emergence of negative refraction we use the formalism proposed in [8], which is based on the analysis of the variation in the phase of  $T(\omega, k_x)$  (transmission coefficient for a plane wave characterized by the transverse wave number  $k_x$ ) of the metamaterial slab with the incident angle  $\theta_i$ . It is observed that the transmission phase decreases with an increase in the incident angle for the two-sided mushroom structure with an inductive load of 0.2 nH. The calculated transmission angle  $\theta_t$  as a function of incident angle  $\theta_i$  at a frequency of 9 GHz is shown in Fig. 2 (b) (red curve), showing a strong negative refraction. Also, in Fig. 2 (b), we plot the calculated transmission angle for different inductive loads. It can be seen that with an increase in the inductive load, the negative refraction angle increases (in comparison with the SC case) and turns to positive refraction. This is due to the decrease

Metamaterials '2011: The Fifth International Congress on Advanced Electromagnetic Materials in Microwaves and Optics in the plasma frequency ( $f_p \approx 8.2$  GHz for inductive load of 0.6 nH) and the structure exhibits positive refraction when one operates above the plasma frequency.

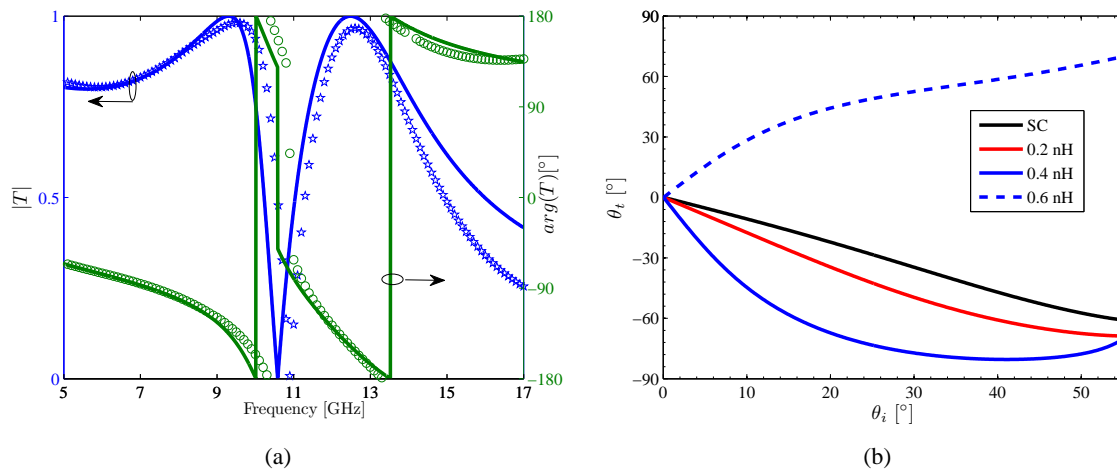


Fig. 2: (a) Comparison of the magnitude and phase of the transmission coefficient for the two-sided mushroom structure excited by a TM-polarized plane wave incident at 60 degrees. The solid lines represent the homogenization results and the symbols correspond to the full-wave HFSS results [7]. (b) Transmission angle  $\theta_t$  as a function of incident angle  $\theta_i$  for the mushroom structure with different inductive loads calculated at the frequency of 9 GHz.

#### 4. Conclusion

The transmission properties of the two-sided mushroom structure have been studied using the nonlocal homogenization model with generalized ABC's at the insertion of impedance loadings and are compared with the full-wave results. It is observed that the mushroom structure exhibits a strong negative refraction at an interface with air, and it is possible to control the negative refraction angle by varying the value of the inductive load. Using the proposed concept of lumped loads it is possible to have a smaller unit cell at the frequency of operation, without resorting to high permittivity dielectrics.

#### References

- [1] M. G. Silveirinha and A. B. Yakovlev, Negative refraction by a uniaxial wire medium with suppressed spatial dispersion, *Physical Review B*, vol. 81, p. 233105, 2010.
- [2] A. Demetriadou and J. B. Pendry, Taming spatial dispersion in wire metamaterial, *J. Phys.: Condens. Matter*, vol. 20, p. 295222, 2008.
- [3] C. S. R. Kaipa, A. B. Yakovlev, and M. G. Silveirinha, Characterization of negative refraction with multilayered mushroom-type metamaterials at microwaves, *J. Appl. Phys.*, vol. 109, p. 044901, 2010.
- [4] S. I. Maslovski, T. A. Morgado, M. G. Silveirinha, C. S. R. Kaipa, and A. B. Yakovlev, Generalized additional boundary conditions for wire media, *New J. Physics*, vol. 12, p. 113047, 2010.
- [5] O. Luukkonen, M. G. Silveirinha, A. B. Yakovlev, C. R. Simovski, I. S. Nefedov, and S. A. Tretyakov, Effects of spatial dispersion on reflection from mushroom-type artificial impedance surfaces, *IEEE Trans. Microwave Theory Tech.*, vol. 57, pp. 2692-99, 2009.
- [6] S. I. Maslovski and M. G. Silveirinha, Nonlocal permittivity from a quasistatic model for a class of wire media, *Physical Review B*, vol. 80, p. 245101, 2009.
- [7] HFSS: High Frequency Structure Simulator based on Finite Element Method, Ansoft Corporation, <http://www.ansoft.com>.
- [8] M. G. Silveirinha, Broadband negative refraction with a crossed wire mesh, *Physical Review B*, vol. 79, p. 153109, 2009.